Staircase Rook Polynomials and Cayley's Game of Mousetrap

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Proposed running head: Mousetrap

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Abstract

First introduced by Arthur Cayley in the 1850's, the game of Mousetrap involves removing cards from a deck according to a certain rule. In this paper we find the rook polynomial for the number of Mousetrap decks in which at least two specified cards are removed. We also find a new expression for the rook polynomial for the number of decks in which exactly one specified card is removed and give expressions for counts of two kinds of Mousetrap decks in terms of other known combinatorial numbers.

In the mid-1800's Arthur Cayley [4, 5] described a game called *Mousetrap* that is played as follows: A deck contains cards numbered 1 through n. Cards are turned over, one-by-one, and are counted. If a card with the same number as the current count is turned over then it is removed from the deck, and the counting begins again from 1 with the next card. Otherwise, the card is placed on the bottom of the deck and the counting is continued. The game is won if all cards are removed from the deck and lost if the count ever reaches $n+1$. The major questions concerning the game are these: 1) How many ways are there to win an n-card game of Mousetrap? 2) How many permutations of the cards $1, 2, \ldots, n$ result in the removal of exactly k cards?

Mousetrap has proved surprisingly difficult to analyze. Steen [14] found expressions for the number of permutations of n cards in which card j, $1 \leq j \leq n$, is the first card removed, the number of permutations in which card 1 and then card j, $j \neq 1$, are the first two cards removed, and the number of permutations in which card 2 and then card j, $j \neq 2$, are the first two cards removed. Unfortunately, his paper contains some errors. Over one hundred years later these were corrected in apparently independent papers by Mundfrom [10] and Guy and Nowakowski [8]. The latter also found an expression for the number of permutations in which only card j is removed, and they raised some additional questions about the game of Mousetrap. (See also Guy and Nowakowski [9] and Problem E37 of Guy [7].) The questions of Guy and Nowakowski have, in turn, been partially addressed by Bersani [1, 2, 3]. However, the results of all of these authors are still far from answering the major questions.

In this paper we determine the rook polynomial for the number of permutations in which card j is the only card removed and for the number of permutations in which card j followed by card k are the first two cards removed. The first result contains the same information as that obtained by Guy and Nowakowski but is expressed in a more compact form. The second result is the major result in the paper, as it extends the work on Mousetrap to the general case of the removal of the first two cards. Finally, we discuss two sets of numbers arising in the study of Mousetrap that are closely related to other known combinatorial numbers.

1 Staircase rook polynomials

Analyzing a specific Mousetrap scenario involves determining a number of permutations subject to a set of restrictions. Rook polynomials are often used for tasks of this kind, as any problem involving permutations with restricted positions can be expressed in terms of rook polynomials [11, p. 165]. Let B be an $n \times m$ chessboard representing an arrangement of n objects into m positions, $n \leq m$, with the property that cell (i, j) is restricted if object i cannot appear in position j. Let $r_i(B)$ be the number of ways of placing i non-attacking rooks on restricted cells in B, with $r_0(B) = 1$. The *rook polynomial* for an $n \times m$ board is the polynomial $R(x, B) = \sum_{i=0}^{n} r_i(B)x^i$.

We make use of two properties of rook polynomials.

Lemma 1. *If there are n objects and n positions, so that B represents permutations with restricted positions, then the number of such permutations is given by $\sum_{i=0}^{n}(-1)^i r_i(B)(n-i)!$.*

Lemma 1 is a consequence of the principle of inclusion and exclusion [12, p. 113]. Its importance for our work is that the rook polynomial for a particular Mousetrap scenario contains enough information to determine the number of permutations in that scenario.

Lemma 2. *If the board B contains subboards B_1 and B_2 such that B_1 and B_2 share no rows or columns and the cells in B not in B_1 or B_2 are unrestricted then $R(x, B) = R(x, B_1)R(x, B_2)$.*

Lemma 2 is a standard result on rook polynomials [12, p. 113].

The rook polynomials for the Mousetrap scenarios we consider can be expressed in terms of a particular class of rook polynomials, the staircase rook polynomials. Specifically, the n^{th} *staircase rook polynomial* $L_n(x)$ is the rook polynomial with n cell restrictions arranged in the staircase pattern in Table 1, where there are $n/2$ rows

$$\begin{array}{cccccc}
\mathrm{X} & \mathrm{X} & & & & \\
 & \mathrm{X} & \mathrm{X} & & & \\
 & & \mathrm{X} & \mathrm{X} & & \\
 & & & \ddots & \ddots & \\
 & & & & \mathrm{X} & \mathrm{X}
\end{array}$$

Table 1: Restricted positions for even staircase rook polynomial

and $n/2 + 1$ columns, in the case in which n is even, or in Table 2, where there are

$$\begin{array}{ccccc}
\mathrm{X} & & & & \\
\mathrm{X} & \mathrm{X} & & & \\
 & \mathrm{X} & \mathrm{X} & & \\
 & & \ddots & \ddots & \\
 & & & \mathrm{X} & \mathrm{X}
\end{array}$$

Table 2: Restricted positions for odd staircase rook polynomial

$(n+1)/2$ rows and $(n+1)/2$ columns, in the case in which n is odd. (Rotations and reflections of these patterns produce identical rook polynomials.)

Riordan [11, pp. 182–183] shows that the n^{th} staircase rook polynomial is of the form $L_n(x) = \sum_{i=0}^{m} \binom{n+1-i}{i} x^i$, where $m = \lfloor (n+1)/2 \rfloor$. The coefficients of $L_n(x)$ are thus the numbers on a shallow diagonal of Pascal's triangle, beginning with $\binom{n+1}{0}$. Moreover, they are known to sum to F_{n+2}, the $n+2$ Fibonacci number [12, p. 104].

2 Analyzing Mousetrap positions

First, some notation. Let $M_{n,j}$ be the number of n-card Mousetrap decks in which card j is the only card removed, and let $R_{n,j}$ be the corresponding rook polynomial. Similarly, let $M_{n,j,k}$ be the number of n-card Mousetrap decks in which card j followed by card k are the first two cards removed; let $R_{n,j,k}$ be the corresponding rook polynomial.

As an illustration of our use of staircase rook polynomials in analyzing Mousetrap, consider the number of 8-card decks in which card 6 is the only card removed. This means that card 6 falls in position 6, and the remaining 7 positions have restrictions according to the board in Table 3.

	Position						
Card	1	2	3	4	5	7	8
1	X					X	
2		X					X
3	X		X				
4		X		X			
5			X		X		
7					X		
8						X	

Table 3: Restricted positions when card 6 is the only card removed from an 8-card deck

At first glance, determining the rook polynomial for this board does not appear to be easy. However, after rearranging rows and columns in the right fashion the pattern shown in Table 4 emerges.

At this point we can see, via Lemma 2, that $R_{8,6}(x)$ can be expressed as the product of two staircase rook polynomials: $R_{8,6}(x) = L_8(x)L_4(x)$. The reasoning behind arranging the rows and columns in this fashion is the following: There is one position in which neither card 8 nor card 1 can appear, one position in which neither 1 nor 3 can appear; one position in which neither 3 nor 5 can appear, and one position in which neither 5 nor 7 can appear. In addition, there is one position in which neither 2 nor 4 can appear. This means that there are two distinct chains of cards – 8,1,3,5,7 and 2,4 – with the property that for any two consecutive cards in the chain there is a position in which those two cards are precisely the cards that cannot appear in that position. But this property simply means that the rook polynomial corresponding to a chain is a

	Position						
Card	7	1	3	5	8	2	4
8	X						
1	X	X					
3		X	X				
5			X	X			
7				X			
2					X	X	
4						X	X

Table 4: After rearranging rows and columns

staircase rook polynomial. To determine which staircase rook polynomials correspond to these chains we simply note that each number appearing in one of these chains that is less than 6 has two restrictions and each number greater than 6 has one. The number of chains is $8 - 6$, or 2. This idea of dividing cards into distinct chains, each of which corresponds to a staircase rook polynomial, is the key idea behind our results.

Theorem 1.

$$R_{n,j} = \begin{cases} \prod_{i=1}^{n-j} L_{\{1+2\lceil (j-i)/(n-j)\rceil - [i\equiv j \bmod (n-j)] + [i=1]\}}, & j < n; \\ L_1^{n-1}, & j = n. \end{cases}$$

Proof. First, suppose $j = n$. The only restrictions are that card i, $1 \le i \le n-1$, cannot appear in position i, as the second set of restrictions appearing in the case $j < n$ is redundant in the case $j = n$. Thus each chain has only one card in it, and each card has only one restriction.

Now suppose $j < n$, and fix i, $1 \le i \le n - j$ and $i \ne j$. There is one main case and two exceptions. We first describe the main case. Card i cannot appear in position $j + i$ or position i. Card $i + (n - j)$ cannot appear in position i or position $i + (n - j)$. Card $i + 2(n - j)$ cannot appear in position $i + (n - j)$ or position $i + 2(n - j)$. This chain continues until we reach the largest card $i + c(n - j)$ strictly less than n. All cards in this chain have two positions in which they cannot appear, except for the final card $i + c(n - j)$, which is necessarily between $j + 1$ and $n - 1$, inclusive, and there is only one such card between these two numbers. (It is possible to have $c = 0$, so that card i forms a chain by itself.) Thus the staircase rook polynomial corresponding to the main case has index equal to twice the number of positive integers less than j that are equivalent to i mod $(n - j)$ plus one for the final card in the chain between $j + 1$ and $n - 1$. This is $1 + 2\lceil (j - i)/(n - j)\rceil$.

One exception is the case $i = 1$, in which card n as well as card 1 cannot appear in position $j + 1$. This adds 1 to the index on the staircase rook polynomial in the $i = 1$ case. The other exception is the case $i \equiv j \bmod (n - j)$, as card j cannot appear in a chain. In this case the final card in the chain of restrictions is $j-(n-j)$, if this expression is greater than 0, or the chain is empty. Either way, the fact that card j is not in the chain means that the index on the staircase rook polynomial in the $i \equiv j \bmod (n-j)$ situation is 1 less than that occurring in the main case. If $1 \equiv j \bmod (n-j)$, then the modifications from these two special cases cancel each other out. $\square$

To use Theorem 1 to determine the number of n-card Mousetrap decks in which card j is the only card removed we calculate $R_{n,j}(x)$ via the theorem, expand it as a polynomial in x, substitute $(-1)^i(n-1-i)!$ for x^i (using Lemma 1), and evaluate. (We substitute $(n-1-i)!$ rather than $(n-i)!$ because the removal of card j means that we are effectively counting permutations on $n-1$ elements rather than on n elements.) For example, $R_{8,6}(x) = L_8(x)L_4(x) = (1+8x+21x^2+20x^3+5x^4)(1+4x+3x^2) = 1+12x+56x^2+128x^3+148x^4+80x^5+15x^6$. Thus the number of 8-card Mousetrap decks in which card 6 is the only card removed is $1(7!) - 12(6!) + 56(5!) - 128(4!) + 148(3!) - 80(2!) + 15(1!) = 791$.

Some specific instances of Theorem 1 have especially nice forms. See, for example, Table 5.

Case	$R_{n,j}$	Expanded in x
$j=1$ or $j=n$	$(L_1)^{n-1}$	$(1+x)^{n-1}$
$2 \le j \le n/2$	$(L_1)^{n-2j}(L_3)^{j-2}L_4$	$(1+x)^{n-2j}(1+3x+x^2)^{j-2}(1+4x+3x^2)$
n odd, $n \ge 3$, $j=(n+1)/2$	$(L_3)^{(n-1)/2}$	$(1+3x+x^2)^{(n-1)/2}$
n even, $n \ge 6$, $j = n/2+1$	$L_2(L_3)^{n/2-3}L_6$	$(1+2x)(1+3x+x^2)^{n/2-3}(1+6x+10x^2+4x^3)$
n odd, $n \ge 3$, $j=n-2$	$(L_{n-2})^2$	
n even, $n \ge 4$, $j=n-2$	$L_{n-4}L_n$	
$n \ge 3, j=n-1$	L_{2n-3}	

Table 5: Some special cases of Theorem 1

Using Theorem 1 we can determine the total number of n-card Mousetrap decks in which exactly one card is removed as well. Because of the linearity of the rook polynomial evaluation, summing the rook polynomials for fixed n over j produces a polynomial that, while not technically a rook polynomial, can be evaluated like a rook polynomial. For example, $\sum_{j=1}^{5} R_{5,j}(x) = (L_1(x))^4 + L_1(x)L_4(x) + (L_3(x))^2 + L_7(x) + (L_1(x))^4 = (1+x)^4 + (1+x)(1+4x+3x^2) + (1+3x+x^2)^2 + (1+7x+15x^2+10x^3+x^4) + (1+x)^4 = 5+26x+45x^2+27x^3+4x^4$. Thus the total number of 5-card decks in which exactly one card is removed is $5(4!) - 26(3!) + 45(2!) - 27(1!) + 4 = 31$.

The method of Theorem 1 can also be used to determine the rook polynomial for the number of n-card Mousetrap decks in which j is the first card removed and k is the second card removed. First, we note that if n is the first card removed then it will be the only card removed. (Suppose card k, $1 \le k \le n-1$, is the second card removed. Then it would have to be in position k. However, if it were in position k it would have been removed before card n, in the first pass through the deck. Thus $M_{n,n,k} = 0$ for any k.) Otherwise, the situation is as described in Theorem 2.

Theorem 2. *For $j < n$, $j \neq k$, we have*

$$R_{n,j,k} = \begin{cases} (L_1)^{k-j-1}(L_2)^{j-1}, & j+k \leq n, j < k; \\ (L_1)^{j-k-1}(L_2)^{k-1}, & j+k \leq n, j > k; \\ \prod_{i=1}^{n-j} L_{\{1+2\lceil (j-i)/(n-j)\rceil - [i \equiv j \bmod (n-j) \text{ or } i=1]\}}, & k = n; \\ \prod_{i=1}^{n-j} L_{\{\lceil (k-i)/(n-j)\rceil + \lceil (j-i)/(n-j)\rceil - [i \equiv j \bmod (n-j)] - [i \equiv k \bmod (n-j)]\}}, & j+k > n, j < k \neq n; \\ (L_1)^{j-k-1} \prod_{i=1}^{n-j} L_{\{2\lceil (k-i)/(n-j)\rceil - [i \equiv k \bmod (n-j)]\}}, & j+k > n, j > k. \end{cases}$$

Proof. Case 1: $j + k \leq n$. In this case no position has more than one card that cannot appear in it. Thus every chain of restricted cards contains only one card. If $j < k$, then cards 1 through $j-1$ have two positions in which they cannot appear, and cards $j+1$ through $k-1$ have one position in which they cannot appear. Thus $R_{n,j,k} = (L_1)^{k-j-1}(L_2)^{j-1}$. If $k < j$, then a similar argument shows that $R_{n,j,k} = (L_1)^{j-k-1}(L_2)^{k-1}$.

Case 2: $k = n$. This case is exactly that in Theorem 1, except that position $j+1$ now contains card n rather than not being able to contain either card n or card 1. Thus the modification to the chain containing 1 is now -1 (as card 1 has one fewer restrictions) rather than $+1$ (the restriction for card n). The one exception occurs when $j = 1$, in which modifying -1 both for $i \equiv j \bmod (n-j)$ and for $i = 1$ is double-counting.

Case 3: $j + k > n$, $j < k$, $k \neq n$. This case is also similar to that of Theorem 1. However, the chain $i, i+(n-j), i+2(n-j), \ldots, i+c(n-j)$ ends with the largest card strictly less than k rather than that strictly less than n. Thus all cards less than j in this chain have two positions in which they cannot appear, and all cards between $j+1$ and $k-1$, inclusive, have one position in which they cannot appear. Thus the index of the staircase rook polynomial containing card i is $\lceil (k-i)/(n-j)\rceil + \lceil (j-i)/(n-j)\rceil - [i \equiv j \bmod (n-j)]$. The one exception is the case $i \equiv k \bmod (n-j)$. In this case (as in the case $i \equiv j \bmod (n-j)$ discussed in Theorem 1), card $k-(n-j)$ is the final card in the chain of restrictions. This card is necessarily smaller than j and so normally would have two positions in which it cannot appear. However, one of these positions is occupied by card k, and so there is actually only one position in which card $k-(n-j)$ cannot appear. (Card n has no restrictions on it; thus, unlike Theorem 1, there is no $+1$ modification in the case $i = 1$.)

Case 4: $j + k > n$, $j > k$. This case is similar to the previous case, with the roles of j and k swapped. However, having $k < j$ means that any cards larger than k are cut off from affecting the chains they do in the previous case. Thus no chains have modifications for card j, and each card between $k+1$ through $j-1$, inclusive, forms a chain by itself. Other than that, chains form as in the previous case, with each card smaller than k having two position restrictions on it and the situation in which $i \equiv k \bmod (n-j)$ having the usual -1 modification. □

For example, $R_{8,6,4}(x) = L_1(x)L_4(x)L_1(x) = (1+x)^2(1+4x+3x^2) = 1+6x+12x^2+10x^3+3x^4$. Thus the number of permutations of 8 cards in which card 6 is the first card removed and card 4 is the second is $1(6!) - 6(5!) + 12(4!) - 10(3!) + 3(2!) = 234$. (Since cards 6 and 4 have their positions fixed we are effectively considering permutations on 6 elements rather than on 8.)

As with Theorem 1, the total number of n-card Mousetrap decks in which at least two cards are removed can be found by determining the polynomial $\sum_{j=1}^{n-1} \sum_{k=1,k\neq j}^{n} R_{n,j,k}$ via Theorem 2 and then evaluating it like a rook polynomial for permutations of $n-2$ elements.

Incidentally, using Theorems 1 and 2 we found minor errors in tables of numbers given in Mundfrom [10] and Guy and Nowakowski [8]. Mundfrom's Table 1 has the number of permutations of 8 cards in which card 2 is the first removed and card 7 is the second removed as 310. However, our formula and our computer simulations for this number both give 309. (Incidentally, evaluating either Mundfrom's expression or Guy and Nowakowski's expression for this number gives 309 as well.) Guy and Nowakowski's extension to their Table 3 has the number of permutations of 17 cards in which card 1 is the only card removed as 76,970,642,511,745. Our formula gives 7,697,064,251,745, so that the digit 1 occurs once rather than twice. (This number is also the number of derangements of 16 elements and so can easily be verified; e.g., Sloane [13].) Again, these are minor mistakes — a miscount by 1 and an apparent typographical error.

3 Two special cases

It is fairly easy to see, by considering the pattern of card restrictions for the different positions, that $M_{n,1} = M_{n,n} = D_{n-1}$, the number of derangements of $n-1$ elements. We consider two other special cases, $M_{n,2}$ and $M_{n,n-1}$, that also have expressions in terms of known combinatorial numbers.

To obtain our expression for $M_{n,2}$ we need two properties of a certain set of numbers. Let $a_{n,i}$, $1 \leq i \leq n$, be the number of permutations of n cards in which card i is the first card removed. Let $a_{n,0}$ be the number of permutations of n cards in which no cards are removed; thus $a_{n,0} = D_n$, the number of derangements of n elements. (These numbers are discussed in both Mundfrom [10] and Guy and Nowakowski [8].) By examining the restricted positions, it is easy to see that $a_{n,i}$ is equal to the number of permutations of $n-1$ elements in which $i-1$ specific elements each have a distinct position in which they do not appear. Two properties of these numbers that we make use of are the following.

Lemma 3. *[14, p. 232]* $a_{n,i} = a_{n,i-1} - a_{n-1,i-1}$, $1 \leq i \leq n$.

Lemma 4. $a_{n,n} = D_{n-1}$.

Lemma 3 is mentioned in both Mundfrom [10] and Guy and Nowakowski [8]; Lemma 4 should be clear.

Theorem 3. *If* $n \geq 4$, $M_{n,2} = D_{n-1} - D_{n-2} - 2D_{n-3}$.

Proof. A permutation counted by $M_{n,2}, n \geq 4$, is characterized by the fact that card 1 cannot appear in positions 1 or 3; card i, $3 \leq i \leq n-2$, cannot appear in position $i+2$; card $n-1$ cannot appear in position 1; and card n cannot appear in position 3. To count these permutations we condition on the placement of card 1. If card 1 appears in position 4 (the only position with no restrictions), then there is a one-to-one correspondence between the remaining cards and positions: Each card has a unique position in which it cannot appear. Thus the number of permutations in which card 2 is in position 2 and card 1 is in position 4 is D_{n-2}. If card 1 is in position i, $5 \leq i \leq n$, then card $i-2$ has no restrictions among the remaining positions, any remaining cards can appear in position 4, and for each card other than $i-2$ there is a unique position in which it cannot appear. Thus the number of permutations in which card 2 is in position 2 and card 1 is in position i is the number of permutations on $n-2$ elements in which $n-3$ specific elements each have a distinct position in which they do not appear: $a_{n-1,n-2}$. Since there are $n-4$ choices for i we have $M_{n,2} = D_{n-2} + (n-4)a_{n-1,n-2}$.

However, Lemmas 3 and 4 yield $a_{n-1,n-2} = a_{n-1,n-1} + a_{n-2,n-2} = D_{n-2} + D_{n-3}$. Thus we have $M_{n,2} = (n-3)D_{n-2} + (n-4)D_{n-3}$. Using a little algebra and Euler's recursive relation for the derangement numbers, $D_n = (n-1)(D_{n-1} + D_{n-2})$ [11, p. 60], produces $M_{n,2} = D_{n-1} - D_{n-2} - 2D_{n-3}$. $\square$

Our expression for $M_{n,n-1}$ involves combining a special case of Theorem 1 with observations and results from Riordan [11, pp. 195–198]. The well-known ménage problem [6, p. 140–142] entails determining the number of ways to seat n married couples around a circular table, alternating male and female, so that no person is sitting next to his or her spouse. The *ménage numbers* $\{u_n\}_{n=0}^{\infty}$, beginning $1, 0, 0, 1, 2, 13, 80, 579, \ldots$, are often used in expressing the solution to this problem. Riordan calls these the *circular ménage numbers*, and considers them together with *straight-table ménage numbers* $\{v_n\}_{n=0}^{\infty}$, beginning 1, 0, 0, 1, 3, 16, 96, 675, $\ldots$, which arise in the solution to the corresponding problem involving a straight table.

Theorem 4. *If* $n \geq 2$, $M_{n,n-1} = v_{n-1} = \sum_{i=1}^{n-1} u_i$.

Proof. Riordan discusses the fact that the rook polynomial for the straight-table ménage number v_n is the staircase rook polynomial L_{2n-1}. By Theorem 1 and Table 5, then, $M_{n,n-1} = v_{n-1}$. Riordan also shows that $u_i = v_i - v_{i-1}$, for $i \geq 2$. Given that $u_1 = v_1 = 0$, this means that $\sum_{i=1}^{n-1} u_i = v_{n-1}$. $\square$

4 Final observations

There are still many open questions about Mousetrap. In particular, the work described in this paper is still far from answering the two questions posed in the first paragraph: How many ways are there to win an n-card game of Mousetrap? How many permutations of the cards $1, 2, \ldots, n$ result in the removal of exactly k cards? Guy and Nowakowski [8] ask several additional questions about Mousetrap, too. (These questions appear in Guy and Nowakowski [9] and Guy [7] as well.) Their questions have yet to be answered definitively, either, although Bersani [1, 2, 3] has obtained some partial results.

We end with the following table of the $R_{n,j}$ rook polynomials. There appear to be relationships among the indices not indicated by Theorem 1 or in Table 5. Is there a way to use these to express the $R_{n,j}$'s in a simpler form than that given in Theorem 1? Similarly, is there a way to express the $R_{n,j,k}$'s in a simpler form than that given in Theorem 2?

$n\backslash j$	1	2	3	4	5	6	7
1	L_1						
2	$-$	$-$					
3	$(L_1)^2$	L_3	$(L_1)^2$				
4	$(L_1)^3$	L_4	L_5	$(L_1)^3$			
5	$(L_1)^4$	L_1L_4	$(L_3)^2$	L_7	$(L_1)^4$		
6	$(L_1)^5$	$(L_1)^2L_4$	L_3L_4	L_2L_6	L_9	$(L_1)^5$	
7	$(L_1)^6$	$(L_1)^3L_4$	$L_1L_3L_4$	$(L_3)^3$	$(L_5)^2$	L_{11}	$(L_1)^6$
8	$(L_1)^7$	$(L_1)^4L_4$	$(L_1)^2L_3L_4$	$(L_3)^2L_4$	$L_2L_3L_6$	L_4L_8	L_{13}
9	$(L_1)^8$	$(L_1)^5L_4$	$(L_1)^3L_3L_4$	$L_1(L_3)^2L_4$	$(L_3)^4$	$L_2L_5L_6$	$(L_7)^2$
10	$(L_1)^9$	$(L_1)^6L_4$	$(L_1)^4L_3L_4$	$(L_1)^2(L_3)^2L_4$	$(L_3)^3L_4$	$L_2(L_3)^2L_6$	$(L_5)^3$
11	$(L_1)^{10}$	$(L_1)^7L_4$	$(L_1)^5L_3L_4$	$(L_1)^3(L_3)^2L_4$	$L_1(L_3)^3L_4$	$(L_3)^5$	$L_2L_3L_5L_6$

$n\backslash j$	8	9	10	11
8	$(L_1)^7$			
9	L_{15}	$(L_1)^8$		
10	L_6L_{10}	L_{17}	$(L_1)^9$	
11	$L_4L_5L_8$	$(L_9)^2$	L_{19}	$(L_1)^{10}$

Table 6: Rook polynomials $R_{n,j}$

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