A Listing of Some Mathematical Notation and Terminology

Note: This document is a work in progress. If you come across mathematical notation that you don't understand, please let me know and I will add it.

Terms

Counterexample A *counterexample* is an example that disproves a claim. A counterexample must provide an actual example. For example, consider the following claim: *Every set of* 5 *numbers must contain the number* 0.

Counterexample: Consider the set $\{1, 2, 3, 4, 5\}$. This is a set of 5 numbers however it does not contain the number 0. Therefore the claim is not true.

Non-negative This phrase means the quantity is greater than or equal to 0. In other words, it is not negative but it could be zero or positive.

Proof by contradiction A *proof by contradiction* is a proof technique. A proof by contradiction begins by assuming the proposition under consideration is false and then derives some contradiction, thus showing the proposition must have been true in the first place.

Transitive A relation over a set of elements (which we'll denote by $r(x, y)$) is called *transitive* if $r(x, y)$ and $r(y, z)$ implies that $r(x, z)$. In other words, if x is related to y and y is related to z then x is related to z. A good example of this is the relation "less than". If $x < y$ and $y < z$ then it must be that $x < z$.

Without loss of generality This phrase can be abbreviated as WLOG and is followed by an assumption that restricts the proof to a special case. "Without loss of generality" means that it is okay to restrict the proof to a special case because the proof of the other cases would follow in the same manner anyways.

Symbols

$\in$ The symbol $\in$ can be read as "is contained in". For example, $x \in A$ means that the element x is contained in the set A.

$\subset$ The symbol $\subset$ means that a set is contained inside of a larger set – i.e. it is a *subset* of a larger set. For example, if A is the set $\{2, 4, 6, 8\}$ then one possible subset is $\{4, 6\}$. We would write this as $\{4, 6\} \subset A$.

$n!$ The expression $n!$ can be read as "n factorial". The factorial of a number n is defined as

$$n! = n * (n - 1) * (n - 2) \ldots 3 * 2 * 1$$

$\sum$ The symbol $\sum$ represents a summation of terms. The bounds of the summation are often written below and above the symbol. For example, the summation

$$\sum_{i=1}^{10} 2 * i$$

is a summation that starts at 1 and ends at 10. For each value of i, we are adding $2 * i$ to the total sum – i.e, this summation is equivalent to $2 + 4 + \ldots + 20$.

$|A|$ The vertical bars $|\ |$ represent the *cardinality* of a set. The cardinality of a set is the number of elements in the set. For example, the set $A = \{a, b, c\}$ has cardinality 3. We denote this by writing $|A| = 3$.

Lists and Sets

$\boldsymbol{X} = \{x_1, \ldots, x_n\}$
X is a set that contains n elements. Those elements can be enumerated using the subscripts x_1, x_2, etc.

$X \times Y$
$X \times Y$ is called the *cartesian product* of X and Y. The cartesian product is also a set. It is the set of all pairs (x, y) where $x \in X$ and $y \in Y$. For example, if $X = \{1, 3\}$ and $Y = \{2, 4\}$ then the cartesian product $X \times Y$ is the set $\{(1, 2), (1, 4), (3, 2), (3, 4)\}$.

$\boldsymbol{x_i, x_j \in X}$ **where** $\boldsymbol{i \neq j}$
This statement says that x_i and x_j are 2 *distinct* elements contained in the set X. In other words, x_i cannot be the same element as x_j.

Functions

$\mathbf{f}(\boldsymbol{x})$, $\boldsymbol{T(n)}$, etc.
This notation is used to denote a *function.* A function is a relation that uniquely assigns each element in the input to an element of the output. For example, $f(x) = 2.5x$ is a function that takes a real-valued number x (i.e. an input) and assigns it the real-valued number that is 2.5 times as big (i.e. the output).

Sometimes you will just see the notation $f(x)$ or $T(n)$ by itself. In this case, it is being used to represent some generic function. Traditionally, x is used to represent a real-valued number whereas n is used to represent an integer-valued number.

$\boldsymbol{\lim_{n \to \infty} h(n) = c}$
This notation represents a *limit.* This means that as the value of n approaches infinity (i.e. denoted as $n \to \infty$), the value of $h(n)$ approaches the number c. A formal definition for a limit is as follows:

$$\lim_{n \to \infty} h(n) = c \text{ if for every } \epsilon > 0 \text{ there exists an } n_0 \text{ such that}$$

$$|h(n) - c| < \epsilon \quad \text{for all } n > n_0$$