HW 1A: Assymptotic Analysis

1. For this question, consider the following program running times: n^2, n^3, $10000n^2$, $n \log n$, 2^n, 2^{2^n}

- **A.** If you doubled the input size n, how much slower would each algorithm get?
- **B.** If you added 1 to the input size n, how much slower would each algorithm get?
- **C.** Assume that the running time above is the *exact* number of instructions required, and that your computer can run 10^{10} instructions per second. What is the largest value of n for which each program would terminate in an hour or less?

2. For this question, arrange the functions in order of ascending growth rate. (That is, if you list $\mathrm{f}(n)$ before $\mathrm{g}(n)$, it must be the case that $\mathrm{f}(n) \in \mathrm{O}(\mathrm{g}(n))$.

$\mathrm{f}_1(n) = n^{2.5}$, $\mathrm{f}_2(n) = \sqrt{2n}$, $\mathrm{f}_3(n) = \log n$, $\mathrm{f}_4(n) = 10^n$, $\mathrm{f}_5(n) = 100^n$, $f_6(n) = n^2 \log n$

3. Certain holiday songs (like "The Twelve Days of Christmas" or "Khad Gadya") add a new line each verse. What is the Θ class of these songs? Explain.

4. Consider searching a binary-search tree for a certain value, that may or may not be in it.

- **A.** Prove that in the *worst case*, searching the BST is $\in \Theta(n)$.
- **B.** Prove that in the *best case*, searching the BST is $\in \Theta(1)$.
- **C.** For general BST search (that is, where the conditions could be best-case, worst-case, or somewhere in between), what are the tightest O and Ω bounds possible?
- **D.** Does general BST search have a Θ class? Why or why not?

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