

Honors 213

Final Exam

Name _____

Monday, May 7, 2007
4:00 PM
200 points

I. Logic and foundations

a. (10 pts.) Construct a truth table to demonstrate that $(p \wedge (q \vee r)) \equiv ((p \wedge q) \vee (p \wedge r))$ (you will need eight rows)

b. (10 pts.) What is a proof?

c. (10 pts.) What is an axiom? Give two distinct points of view on this.

d. (10 pts.) What are the components of an axiomatic system? Illustrate your answer by giving examples of these components drawn from our study of geometry.

- e.. (5 pts.) What does it mean for an axiom system to be categorical? Is Euclidean geometry categorical?
- f. (15 pts.) What is the Metamathematical Theorem and how do we go about proving it?

g. (10 pts.) Give brief definitions of **intuitionism** and **formalism**.

h. (10 pts.) How do we prove something by using a *reductio ad absurdum* proof?

i. (10 pts.) What does it mean to say that an axiom is independent of a given set of axioms? Give an example we have looked at this term.

j. (10 pts.) Briefly describe the Poincaré model of hyperbolic geometry, giving interpretations of the undefined terms.

- k (10 pts.) Describe how we know that IA 1 holds in the Poincaré model. You can proceed by describing the construction if you like (don't actually do it).

II. Geometry

- a. (15 pts.) Using a careful statement/justification format, prove that for every line there exists a point not on that line. (Prop 2.3)

- b. (15 pts.) Do one (but only one) of the following:
1. Given a triangle ABC there is at least one vertex from which the perpendicular dropped from that vertex to the opposite side meets the opposite side in a point between the two endpoints of the opposite side.
 2. The sum of the degree measures of any two angles in a triangle is less than or equal to the degree measure of their remote exterior angle (corr. 1 to the Saccheri-Legendre theorem)
 3. If triangles are without defect, then a triangle inscribed in a semi-circle is a right triangle.
 4. Prove the Universal Hyperbolic Theorem.

III. Some History

1. (20 pts.) Write brief paragraphs on two of the following persons saying what their contribution to geometry was:

Farkas Bolyai
János Bolyai
Clairaut
Dedekind
Euclid
Frege

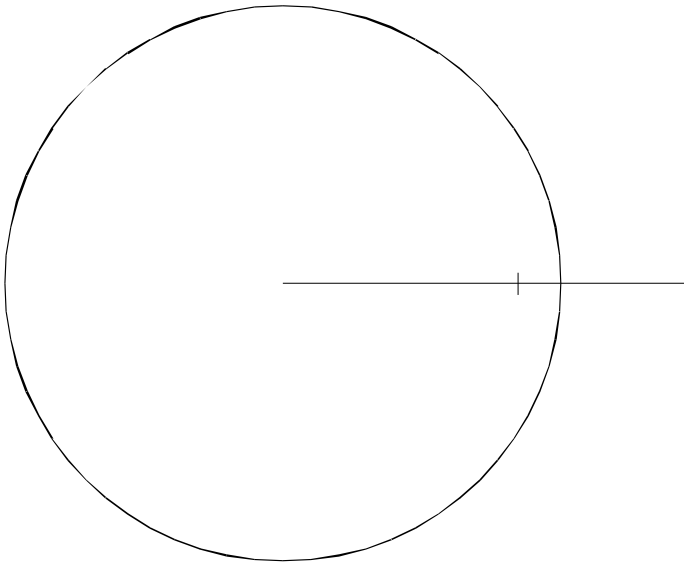
Gauss
Hilbert
Klein
Lambert
Laplace
Legendre

Lobachevsky
Poincaré
Proclus
Saccheri
Thales

IV Final questions and constructions.

1. (10 pts.) What, in your opinion, is the most beautiful theorem in this course? Why?

2. (15 pts.) Give a definition of the inverse of a point and, using a straight-edge and compass, construct the inverse of the point indicated by the cross-hairs in the following diagram. Why is the inverse important?



3. (15 pts. In the Klein model for hyperbolic geometry, construct the common perpendicular to the two lines below:

