Honors 213

Second Hour Exam

Name ________________________________

Friday, March 23, 2007
90 points (will be adjusted to 100 pts. in the gradebook)

I. Some definitions (5 points each). Give formal definitions of the following:

a. Line separation property

b. Congruence of triangles

c An affine geometry

d. A projective geometry

e. Point P is in the interior of angle ABC.

f. The interior of a triangle.

2. (10 pts.) How do we know for sure that we can not construct a proof of the parallel postulate from the three incidence axioms?

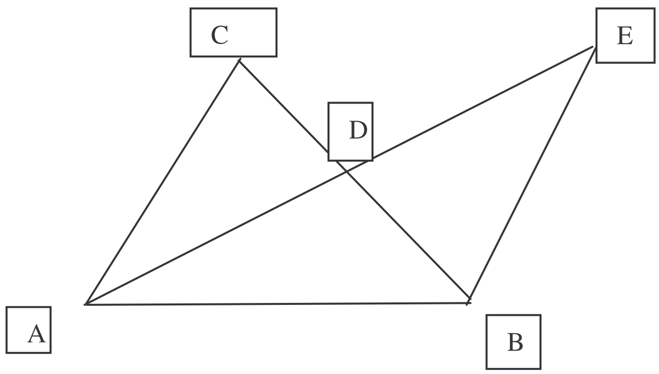
3. (10 pts). Consider the Klein model given in the homework exercises: points are the usual points inside a given circle, lines are chords of the circle (lines running from one point on the circumference of the circle to another, but only points that are actually inside the circle are considered points on a line).

Sketch a picture of the Klein model. Draw a line in the model, select a point in the model not incident with the line just drawn, and draw two lines (in the Klein) through that point and parallel to the given line (recall the definition of parallel lines). What property (elliptic, Euclidean, hyperbolic, none) does this example illustrate?

4. (15 pts.) Suppose that A*B*C and B*C*D. We want to show that A*B*D (i.e., prove part of the corollary to Proposition 3.3). Fill in any justifications for the statements below not already provided:

Statement		Justification
1.	A, B, C, and D are distinct collinear points on a line	(The proof is similar to problem 1 (a) and (b) on page 104)
2.	There is a point E not on this line	
3.	There is a line ***l*** through E and C	
4.	B and D are on opposite sides of the line	
5.	We want to show that A and B are on the same side of this line. Assume, on the contrary, that A and B are on opposite sides of this line.	
6.	Then the segment AB intersects the line through E and C	
7.	That point must be C	
8.	Therefore we have A*C*B	
9.	A*B*C	
10.	Steps 8 and 9 can not both be true.	
11.	Therefore A and B are on the same side of the line through E and C	
12.	Therefore A and D are on opposite sides of the line through E and C.	
13.	The segment AD meets the line through E and C in a point.	
14.	That point must be C	
15.	Therefore A*C*D.	

5. (10 pts.) Given triangles as in the diagram below:



Where AD is congruent to DE, CD is congruent to DB, and angle EDB is congruent to CDA. Sketch a proof (with justifications) that angle EAC is congruent to angle AEB

6. (10 pts.) One of the exercises we did with straight-edge and compass was to make a copy of a triangle.(Major exercise 1.f from chapter 1). We proved that we could do this as a corollary to CA 6 (SAS). Why was it important that we be able to prove that this construction could be done using Hilbert's axioms?

7. (5 pts.) Pick one of the names from the following list and say something appropriate to the course about that person

David Hilbert
Felix Klein
Moritz Pasch

Axioms

(These pages may be removed and need not be returned with the exam. Please make sure that you don't tear off any exam questions!)

EP1: For every point P and for every point Q not equal to P there exists a unique line **l** that passes through P and Q

EP2: For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE.

EP3: For every point O and every point A not equal to O there exists a circle with center O and radius OA.

EP4: All right angles are congruent to each other.

EP5 (Euclidean Parallel Postulate): If two lines are cut by a common transversal making interior angles on one side of the transversal whose degree measures add up to less than 180 degrees, then the two lines meet on that side of the transversal..

LR1: The following are the six types of justifications allowed for statements in proofs:

(1) By hypothesis (given)
(2) By axiom/postulate ...
(3) By theorem ... (previously proved)
(4) By definition ...
(5) By step ... (a previous step in the argument
(6) By rule ... of logic

LR2: To prove a statement H => C, assume the negation of statement C (RAA Hypothesis) and deduce an absurd statement using the hypothesis H if needed in your deduction.

LR3: The statement "not (not S)" means the same thing as "S".

LR4: The statement "not[H=>C]" means the same thing as "H and not C".

LR5: The statement "not[P and Q]" means the same thing as "not P or not Q".

LR6: The statement "not (forall(x) S(x))" means the same thing as "there exists(x) not(S(x))"

LR7: The statement "not (there exists(x) S(x))" means the same thing as "forall(x) not S(x)".

LR8: (modus ponens) If P => Q and P are steps in a proof, then Q is a justifiable step.

LR9:
(a) []P => Q] & [Q => R]] => [P => R].
(b) [P & Q] => P, [P & Q] => Q.
(c) [~Q => ~p] <=> [p => Q]

LR10: For every statement P, "P or ~P" is a valid step in a proof.

LR11: Suppose the disjunction of statements S1 or S2 or ... or Sn is already a valid step in a proof. Suppose that proofs of C are carried out from each of the case assumptions S1, S2, ... Sn. Then C can be concluded as a valid step in the proof (proof by cases).

IA 1: For every point P and for every point Q not equal to P there exists a unique line l incident with P and Q.

IA 2: For every line l there exists at least two points incident with l

IA 3: There exist three distinct points with the property that no line is incident with all three of them.

BA 1: IF A*B*C then A, B, and C are three distinct collinear points, and C*B*A

BA 2: Given any two distinct points B and D, there exist points A, C, and E lying on the line through B and D such that A*B*D, B*C*D, and B*D*E.

BA 3: If A, B, and C are three distinct collinear points, then one and only one of the points is between the other two.

BA4: For every line l and for any three points A, B, and C not lying on l,

- (i) if A and B are on the same side of l and B and C are on the same side of , then A and C are on the same side of l
- (ii) If A and B are on opposite sides of l and B and C are on opposite sides of l, then A and C are on the same side of l
- (corr:) If A and B are on opposite sides of l and B and C are on the same side of l, then A and C are on opposite sides of l

CA1: Copying segments onto rays

CA2: Congruency of segments is an equivalence relation

CA3: Addition of segments

CA4: Copying angles

CA5: Congruency of angles is an equivalence relation

CA6: SAS