Honors 213 / Math 300

Final Exam

Name __________________________________

Friday, May 12, 2006
8:00 AM
200 points

I. Logic and proof

a. (10 pts.) Construct a truth table to demonstrate that $(\square q \square (p \square q)) \equiv \square p$

b. (10 pts.) Simplify the following by moving negations in as far as possible

1. $\square\square x \square y(\square(x, y)$

2. $\square(p \square (p \square q))$

c. (10 pts.) Suppose that we have a series of statements S_i and a conclusion C. What does it mean to say that the conclusion follows logically from the statements S_i? That is, what does it mean to say that C follows logically from $S_1 \wedge S_2 \wedge \cdots \wedge S_n$?

d. (10 pts.) What is

1. An RAA proof?

2. A proof by cases?

II. Interpretations and Models

a. (10 pts.) What is an axiomatic system? How do beer mugs, tables and chairs figure in?

b. (10 pts.) What is an **interpretation** of an axiomatic system? A **model**? How do the two differ?

c. (10 pts.) Briefly describe the Klein and Poincaré models of hyperbolic geometry.

d. (10 pts.) Define **betweeness** in the Klein model. Pick any **betweeness** axiom and demonstrate that this axiom holds in the Klein model.

e. (15 pts.) Outline a proof of the “metamathematical theorem”. As a part of your discussion, say what the corollary to the "MMT" means and why it holds.

III. Neutral Geometry

a. (10 pts. each)

1. Give a definition of an endpoint to a line, and then prove that they can not exist.

2. (problem 15, page 138). Given a triangle ABC. Let D be the midpoint of BC and E the unique point on the line through A and D such that A*D*E and AD congruent to DE. Draw a diagram and show that the angle sum of triangle ABC is the same as triangle AEC.

3. Giving careful justifications for each step, prove the following consequence of theorem 4.1: That given a line **l** and a point **P** not on **l**, there exists at least one line through **P** and parallel to **l**.

IV. Hyperbolic Geometry

a. (10 pts.) Explain why hyperbolic geometry does not permit the existence of rectangles. How does the Greek mathematician Proculus enter into the picture?

b. (15 pts.) In hyperbolic geometry, we have two different kinds of parallel lines. Say what they are and how we can tell them apart. Using either the Klein or Poincaré model of hyperbolic geometry (your choice) illustrate both kinds.

V. Philosophy and History

1. (15 pts.) Write brief paragraphs on two of the following persons saying what their contribution to geometry was:

Farkas Bolyai	Gauss	Lobachevsky
János Bolyai	Hilbert	Poincaré
Clairaut	Klein	Proculus
Dedekind	Lambert	Saccheri
Euclid	Laplace	Thales
Frege	Legendre	

2. (5 pts.) What does it mean to say that the parallel postulate is independent of the other axioms of neutral geometry?

3. (10 pts.) What would be the consequence to mathematics should any attempted proof of the parallel postulate in neutral geometry be successful?

4. (10 pts.) What is the geometry of space? How would Poincaré have responded to this question?

5. (10 pts.) What, in your opinion, is the most beautiful theorem in this course? Why?