Math 300 / Honors 213

Fourth Hour Exam

Name ______________________________

Wednesday, April 26, 2006
95 pts. (will be normalized to
100 pts. in the gradebook

1. (5 pts.) Some definitions and statements of theorems (5 pts. each)

   a, What is a Saccheri quadrilateral?

   b. What is a Lambert quadrilateral?

   c. State the Hilbert Parallel Postulate (being careful not to say more than you need to)

   d. State the Hyperbolic axiom (again being careful not to say more than you need to)

2. (15 pts.) Do one of the following. Please indicate clearly which one you are doing. These propositions (and others about the parallel postulate) can be found in the list of theorems available with this test.

a. Prove: Hilbert's Parallel Postulate <=> [If a line intersects one of two parallel lines it intersects the other] (prop 4.7)

b. Prove: Hilbert's Parallel Postulate <=> [If **t** is a transversal to **l** and **m**, and if **l** is parallel to **m** and if **t** is perpendicular to **l**, then **t** is perpendicular to **m**] (prop 4.9).

3. (5 pts.) Why do we know that in hyperbolic geometry we can not have a triangle whose angle sum is = 180°?

4. (5 pts.) What do we know about similar triangles in hyperbolic geometry?

5. (10 pts.) In Euclidean geometry, if two lines are parallel and are cut by a transversal, then alternate interior angles are congruent. Use this to show that the angle sum of a triangle in Euclidean geometry is 180° (prop 4.11)

The story so far: Although there were many attempts to prove the parallel postulate (remember the student of Gauss who wrote his dissertation on a number of these failed proofs), we will focus our attention here on two who almost had it figured out, and two who figured it out but whose contributions were not recognized until after their deaths, and (finally) consider the ghostly presence of one of the great names of mathematics.

5. (10 pts) Girolamo Saccheri (1667-1733) published *Euclid Freed of Every Blemish*. One of the blemishes was, of course, the parallel postulate. Briefly describe his approach, including a discussion of the three cases he considered for the quadrilaterals named after him. How did he handle the "acute hypothesis"?

6. (10 pts.) Johann Heinrich Lambert (1728 - 1777) was one of the many fine mathematicians the Lorraine has given us. In his efforts to prove the parallel postulate using a *reductio ad absurdum* argument, he came across several surprising results. What was his approach? What did he discover before giving up the chase?

Two mathematicians of the very first rank who almost had hyperbolic geometry in their hands. We next consider two who "got it", but were not very successful in getting the story out in their lifetimes.

7. (10 pts.) Briefly describe the efforts of János Bolyai (1802 - 1860) and Nikolai Lobachevsky (1792 - 1856) in developing what we now call hyperbolic geometry. How did their approach differ from that of Saccheri and Lambert? How were their ideas received?

8. (10 pts.) What role did the great Carl Friedrich Gauss (1777 - 1855) play in this with respect to the work of Bolyai and Lobachevsky? What role did he (Gauss) play in the development of non-Euclidean geometry?

Random comment (more details on Friday):

Who made me the genius I am today,
The mathematician that others all quote?
Who's the professor that made me that way,
The greatest that ever got chalk on his coat?

One man deserves the credit,
One man deserves the blame,
and Nicolai Ivanovich Lobachevsky is his name. (Hey!)

-Tom Lehrer