Math 300 / Honors 213

Third Hour Exam

Name _____

Friday, March 31, 2006
90 points (will be adjusted
to 100 pts in the gradebook)

1. (5 pts.) Some definitions (5 pts. each)

a, AB is less than segment CD (AB < CD).

b. Angle CAB is less than angle EDF

c. Triangle ABC is congruent to triangle DEF

d. A Dedekind cut

c. Neutral Geometry

2. (10 pts.) Dedekind's axiom applies to rays as well as lines. With this in mind, consider the following sets of **positive** numbers on the real number line (the positive numbers form a ray on the real number line): $A = \{x \mid x^2 > 2\}$ and $B = \{x \mid x^2 < 2\}$. Is this a Dedekind cut (and why)? What is the point (a number) guaranteed by Dedekind's axiom? Hint: remember that if c is between a and b then c^2 is between a^2 and b^2

3. Several short proofs (10 pts. each)

a. Show that given segment AB there is an equilateral triangle ABC with all sides congruent to AB.

b. Show that if a triangle is equiangular (all angles are congruent) if and only if it is equilateral.

c. Show that vertical angles are congruent (prop 3.14).

d. Show corollary 1 to the Saccheri-Legendre theorem: that the sum of the degree measures of any two angles in a triangle is less than or equal to the degree measure of their remote exterior angle.

4. Philosophical Musings (10 pts.). Do one of the following:

   a. Say something about the importance of the continuity axioms.
   b. What is the relation between Hilbert's axioms and Euclid's propositions.

5. (5 pts.) Say something about one of the following three names. There is a hitch. It is not sufficient simply to refer to an axiom or theorem named after the person.

a. Hilbert
b. Saccheri
c. Legendre