Honors 213 / Math 300

Second Hour Exam

Name ______________________________

Monday, March 6, 2006
95 points (will be adjusted to
100 pts in the gradebook)

I. Some definitions (5 points each). Give formal definitions of the following:

a. Points P and Q are on the opposite side of line l (do not refer to "same side" in your definition)

b. An equivalence relation

c. The interior of angle ABC

d. Ray $\vec{A}\vec{D}$ is between rays $\vec{A}\vec{B}$ and $\vec{A}\vec{C}$

e. An affine geometry

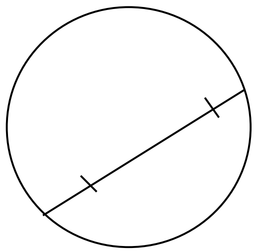
f. A projective geometry

II. Models and the like.

a. (10 pts.) Why can we assert with confidence that there exists no proof of the Euclidean parallel postulate from the three incidence axioms?

b. (10 pts.) Consider the Klein model given in the homework exercises: points are the usual points inside a given circle, lines are chords of the circle (lines running from one point on the circumference of the circle to another, but only points that are actually inside the circle are considered points on a line). Illustrate with brief sketches using this model: (IA 1 is done for you by way of illustration

IA1



IA3

BA1

(problem II b continued

BA2

BA3

The hyperbolic property.

III. Proofs (10 pts. each)

In the following problems, you will be asked to prove statements. Although you should feel free to use axioms, propositions, or corollaries, you should not use problems you remember solving unless the statement says that you are free to do so (proofs of propositions and corollaries excepted, of course!). If you get stuck but remember a problem you did that could help, you may do that for partial credit.

1. Show carefully that if A and B are points then AB = BA. Please recall that both AB and BA are sets.

2. Given that A*B*C, show that AB is a subset of AC. Be careful to consider all cases.

3. Suppose that ray $\vec{A}\vec{D}$ is between rays $\vec{A}\vec{B}$ and $\vec{A}\vec{C}$, and that P is in the interior of angle BAE. Show that P is in the interior of angle BAC. (draw a picture).

IV. Philosophical musings (10 pts.)

While mathematicians do like to be careful, they generally don't like to waste time. Yet much of what we have discussed so far appears to be making a fuss over obvious details. Why be so careful? Why is David Hilbert making such a fuss over things like betweeness, and congruence, and the like? Is there a reason for this?

V. (5 pts.) Pick one of the names from the following list and say something appropriate to the course about that person

David Hilbert
Felix Klein
Moritz Pasch

Axioms and major propositions

EP1: For every point P and for every point Q not equal to P there exists a unique line **l** that passes through P and Q

EP2: For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE.

EP3: For every point O and every point A not equal to O there exists a circle with center O and radius OA.

EP4: All right angles are congruent to each other.

Euclidean Parallel Postulate: For every line **l** and for every point P that does not lie on **l** there exists a unique line m through P that is parallel to **l**.

LR1: The following are the six types of justifications allowed for statements in proofs:
- (1) By hypothesis (given)
- (2) By axiom/postulate ...
- (3) By theorem ... (previously proved)
- (4) By definition ...
- (5) By step ... (a previous step in the argument
- (6) By rule ... of logic

LR2: To prove a statement H => C, assume the negation of statement C (RAA Hypothesis) and deduce an absurd statement using the hypothesis H if needed in your deduction.

LR3: The statement "not (not S)" means the same thing as "S".

LR4: The statement "not[H=>C]" means the same thing as "H and not C".

LR5: The statement "not[P and Q]" means the same thing as "not P or not Q".

LR6: The statement "not (forall(x) S(x))" means the same thing as "there exists(x) not(S(x))"

LR7: The statement "not (there exists(x) S(x))" means the same thing as "forall(x) not S(x)".

LR8: (modus ponens) If P => Q and P are steps in a proof, then Q is a justifiable step.

LR9:
- (a) []P => Q] & [Q => R]] => [P => R].
- (b) [P & Q] => P, [P & Q] => Q.
- (c) [~Q => ~p] <=> [p => Q]

LR10: For every statement P, "P or ~P" is a valid step in a proof.

LR11: Suppose the disjunction of statements S1 or S2 or ... or Sn is already a valid step in a proof. Suppose that proofs of C are carried out from each of the case assumptions S1, S2, ... Sn. Then C can be concluded as a valid step in the proof (proof by cases).

IA 1: For every point P and for every point Q not equal to P there exists a unique line l incident with P and Q.

IA 2: For every line l there exists at least two points incident with l

IA 3: There exist three distinct points with the property that no line is incident with all three of them.

Prop 2.1: Non parallel distinct lines have a unique point in common
Prop 2.2: There exist three distinct non concurrent lines
Prop 2.3: For every line there is at least one point not lying on it.
Prop 2.4: For every point there is at least one line not passing through it.
Prop 2.5 : For every point P there exist at least two lines through P

BA 1: IF A*B*C then A, B, and C are three distinct colinear points, and C*B*A

BA 2: Given any two distinct points B and D, there exist points A, C, and E lying on the line through B and D such that A*B*D, B*C*D, and B*D*E.

BA 3: If A, B, and C are three distinct colinear points, then one and only one of the points is between the other two.

BA4: For every line l and for any three points A, B, and C not lying on l,

(i) if A and B are on the same side of l and B and C are on the same side of , then A and C are on the same side of l

(ii) If A and B are on opposite sides of l and B and C are on opposite sides of l, then A and C are on the same side of l

(corr:) If A and B are on opposite sides of l and B and C are on the same side of l, then A and C are on opposite sides of l

Prop. 3.1: Lines the union of rays AB and BA, and the rays have only the segment AB in common.

Prop 3.2: Every line bounds exactly two half-planes and these half-planes have no point in common.

Prop 3.3: Given A*B*C and A*C*D then B*C*D and A*B*D
corr: Given A*B*C and B*C*D then A*B*D and A*C*D.

Prop 3.4: IF C*A*B and l is the line through A, B, and C, then for every point P lying on l, P lies either on the ray AB or on the opposite ray AC.

Pasch's Theorem: If A, B, C are distinct noncollinear points and l is any line intersecting AB at a point between A and B, then l also intersects AC or BC. If C does not lie on l, then l does not intersect both AC and BC.

Prop. 3.6 Given A*B*C. Then B is the only point common to ray BA and ray BC, and ray AC = ray AC.

Prop 3.7: Given an angle CAB and point d lying on line BC. Then D is in the interior of angle CAB if and only if B*D*C

Prop. 3.8: If D is in the interior of angle CAB then (a) so is every other point on ray AD except a; (b) no point on the opposite ray to AD is in the interior of angle CAB, and (c) if C*A*E then B is in the interior of angle DAE.

Crossbar Theorem: if ray AD is between ray AC and ray AB, then ray AD intersects segment BC.

Prop. 3.9:

(a) if ray r emanating from an exterior point of triangle ABC intersects side AB in a point between A and B, then r also intersects side AC or side BC.

(b) If a ray emanates from an interior point of triangle ABC then it intersects one of the sides, and if it does not pass through a vertex it intersects only one side.