Honors 213

Third Hour Exam

Name ________________________________

Monday, March 27, 2000
100 points

Please note: Because of multiple exams given Monday, this exam will be returned by Thursday, March 30.

1. (5 pts.) Define what it means to say that segment AB is less than segment CD (AB < CD).

2. (15 pts.) Prove the following part of proposition 3.13: that given two segments AB and CD, exactly one of the following is true: AB < CD, AB congruent CD, AB > CD (i.e., CD < AB).

3. (15 pts.) In the proof of proposition 3.16 (for every line l and for every point P there exists a line through P perpendicular to l), we did the case where P was not on line l carefully in class, and then I "hand waved" through the case where P was on l. Here is the rest of the proof. Please supply reasons for each step (this is from the textbook).

There is a point Q not lying on l

We can drop a perpendicular from Q to the line l, obtaining a right angle

We can lay off an angle congruent to this angle with vertex at P and one side on l

The other side of this angle is part of a line through P perpendicular to l.

4. (15 pts.) Define vertical angles and prove that vertical angles are congruent.

5. (10 pts.) Given an informal explanation of why the axioms of continuity are necessary. Illustrate your point using one or more of the continuity axioms (explaining how the axiom applies to your explanation).

6. (10 pts.) In Hardy's view, what is the first test of a work of mathematics?

6. (10 pts.) What two mathematical theorems does Hardy prove in his paper?

7. (20 pts.) Briefly describe how Euclid's and Hilbert's axioms differ, and give an explanation you might give an intelligent fellow student on why Hilbert may have thought his approach necessary.

Axioms and major propositions

EP1: For every point P and for every point Q not equal to P there exists a unique line **l** that passes through P and Q

EP2: For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE.

EP3: For every point O and every point A not equal to O there exists a circle with center O and radius OA.

EP4: All right angles are congruent to each other.

Euclidean Parallel Postulate: For every line **l** and for every point P that does not lie on **l** there exists a unique line m through P that is parallel to **l**.

LR1: The following are the six types of justifications allowed for statements in proofs:
(1) By hypothesis (given)
(2) By axiom/postulate ...
(3) By theorem ... (previously proved)
(4) By definition ...
(5) By step ... (a previous step in the argument
(6) By rule ... of logic

LR2: To prove a statement H => C, assume the negation of statement C (RAA Hypothesis) and deduce an absurd statement using the hypothesis H if needed in your deduction.

LR3: The statement "not (not S)" means the same thing as "S".

LR4: The statement "not[H=>C]" means the same thing as "H and not C".

LR5: The statement "not[P and Q]" means the same thing as "not P or not Q".

LR6: The statement "not (forall(x) S(x))" means the same thing as "there exists(x) not(S(x))"

LR7: The statement "not (there exists(x) S(x))" means the same thing as "forall(x) not S(x)".

LR8: (modus ponens) If P => Q and P are steps in a proof, then Q is a justifiable step.

LR9: (a) []P => Q] & [Q => R]] => [P => R].
(b) [P & Q] => P, [P & Q] => Q.
(c) [~Q => ~p] <=> [p => Q]

LR10: For every statement P, "P or ~P" is a valid step in a proof.

LR11: Suppose the disjunction of statements S1 or S2 or ... or Sn is already a valid step in a proof. Suppose that proofs of C are carried out from each of the case assumptions S1, S2, ... Sn. Then C can be concluded as a valid step in the proof (proof by cases).

IA 1: For every point P and for every point Q not equal to P there exists a unique line l incident with P and Q.

IA 2: For every line l there exists at lease two points incident with l

IA 3: There exist three distinct points with the property that no line is incident with all three of them.

Prop 2.1: Non parallel distinct lines have a unique point in common
Prop 2.2: There exist three distinct non concurrent lines
Prop 2.3: For every line there is at least one point not lying on it.
Prop 2.4: For every point there is at least one line not passing through it.
Prop 2.5 : For every point P there exist at least two lines through P

BA 1: IF A*B*C then A, B, and C are three distinct colinear points, and C*B*A
BA 2: Given any two distinct points B and D, there exist points A, C, and E lying on the line through B and D such that A*B*D, B*C*D, and B*D*E.
BA 3: If A, B, and C are three distinct colinear points, then one and only one of the points is between the other two.
BA4: For every line l and for any three points A, B, and C not lying on l,
- (i) if A and B are on the same side of l and B and C are on the same side of , then A and C are on the same side of l
- (ii) If A and B are on opposite sides of l and B and C are on opposite sides of l, then A and C are on the same side of l
- (corr:) If A and B are on opposite sides of l and B and C are on the same side of l, then A and C are on opposite sides of l

Prop. 3.1: Lines the union of rays AB and BA, and the rays have only the segment AB in common.
Prop 3.2: Every line bounds exactly two half-planes and these half-planes have no point in common.
Prop 3.3: Given A*B*C and A*C*D then B*C*D AND A*B*D
corr: Given A*B*C and B*C*D then A*B*D and A*C*D.
Prop 3.4: IF C*A*B and l is the line through A, B, and C, then for every point P lying on l, P lies either on the ray AB or on the opposite ray AC.
Pasch's Theorem: If A, B, C are distinct noncollinear points and l is any line intersecting AB at a point between A and B, then l also intersects AC or BC. If C does not lie on l, then l does not intersect both AC and BC.
Prop. 3.6 Given A*B*C. Then B is the only point common to ray BA and ray BC, and ray AC = ray AC.
Prop 3.7:Given an an angle CAB and point d lying on line BC. Then D is in the interior of angle CAB if and only if B*D*C
Prop. 3.8: If D is in the interior of angle CAB then (a) so is every other point on ray AD except a; (b) no point on the opposite ray to AD is in the interior of angle CAB, and (c) if C*A*E then B is in the interior of angle DAE.
Crossbar Theorem: if ray Ad is between ray AC and ray AB, then ray AD intersects segment BC.
Prop. 3.9: (a) if ray r emanating from an exterior point of triangle ABC intersects side AB in a point between A and B, then r also intersects side AC or side BC. (b) If a ray emanates from an interior point of triangle ABC then it intersects one of the sides, and if it does not pass through a vertex it intersects only one side.
CA1: Copying segments onto rays
CA2: Congruency of segments is an equivalence relation
CA3: Addition of segments
CS4: Copying angles
CA5: Congruency of angles is an equivalence relation
CA6: SAS
corr: copying triangles
Prop 3.10: if in triangle ABC we have AB congruent to AC, then angle B is congruent to angle C.
Prop 3.11: Segment subtraction.
Prop 3.12: Given segment AC congruent to segment DF. Then for any point B between A and C, there is a unique point E between D and F such that AB is congruent to DE.
Prop 3.13: Segment ordering.
Prop. 3.14: Supplements of congruent angles are congruent.
Prop 3.15: Vertical angles are congruent to each other. An angle congruent to a right angle is a right angle.
Prop. 3.16: For every line l and for every point P there exists a line through P perpendicular to l

Prop 3.17: ASA congruence of triangles.
Prop 3.18: If in triangle ABC we have angle B congruent to angle C, then segment AB is congruent to segment AC, and triangle ABC is isosceles.
Prop 3.19: Angle addition.
Prop 3.20: Angle subtraction
Prop 3.21: Ordering of angles
Prop 3.22: SSS congruence of triangles.
Euclid's Proposition 1: Given any segment, there is an equilateral triangle having the given segment as one of its sides.
Circular Continuity Principle: if a circle C has one point inside and one point outside another circle C', then the two circles intersect in two points.
Elementary Continuity Principle: If one end point of a segment is inside a circle and the other side, then the segment intersects the circle.
Archimedes' Axiom: IF CD is any segment, A any point, and r any ray with vertex A, then for every point B != A on r there is a number n such that when CD is laid off n times on r starting at A, a point E is reached such that nCD is congruent to AT and either B = E or A*B*E.
Aristotle's Axiom: Given any side of an acute angle and any segment AB there exists a point Y on the given side of the angle such that if X is at the foot of the perpendicular from Y to the other side of the angle, XY > AB.
Corr: Let ray AB be any ray, P any point not collinear with A and B, an angle XVY any acute angle. Then there exists a point R on ray AB such that angle PRA < angle XVY.
Dedekind's Axiom: Suppose the set {l} of points on a line l is the disjoint union of two nonempty subsets X and Y such that no point of one is between two points of the other. Then there exists a unique point O on l such that one of the subsets is equal to a ray of l with vertex O and the other set is equal to the complement.
Hilbert's Axiom of Parallelism: For every line l and every point P not lying on l there is at most one line m through P such that m is parallel to l.

(this is the end of the axioms and propositions that will be provided in this class)