Honors 213

Second Hour Exam

Name ________________________________

Thursday, March 2, 2000
100 points

Please note: Because of conference and candidate obligations, this exam will be returned by Thursday, March 9.

I. Some definitions (7 points each). Give formal definitions of the following:

a. Points P and Q are on the same side of line l

b. An equivalence relation

c. The interior of angle ABC

d. A projective geometry

e. In the definition of a segment AB (A, B, and all of the points on the line through AB that lie between A and B, I've complained that part of the definition is unnecessary. What part is it, and why? (Hint: consider the betweeness axioms).

II. Proofs

In the following problems, you will be asked to prove statements. Although you should feel free to use axioms, propositions, or corollaries, you should not use problems you remember solving unless the statement says that you are free to do so (proofs of propositions and corollaries excepted, of course!). If you get stuck but remember a problem you did that could help, you may do that for partial credit.

1. Given that A*B*C, show that AB is a subset of AC. Be careful to consider all cases.

1. In several (informal) discussions, we have used the fact that to show that a point P is in the interior of triangle ABC, it is sufficient to show that it is in the interior of two of the angles of the triangle. Let's show this rigorously. Show that if P is in the interior of angles A and B in triangle ABC, then it is in the interior of angle C. HINT: This one can almost be done by writing down the definitions.

3. In triangle ABC, point D is between A and B, and point E is between A and C. Show that D is on the same side of the line through BC as A is.

1. To demonstrate equality of two sets, it is necessary to show that each is a subset of the other. Use this fact to prove half of the statement that if A*B*C, then the ray AB is equal to the ray AC. Be sure to say which of the two subset statements you are proving.

II. Philosophical musings (20 pts.)

While mathematicians do like to be careful, they generally don't like to waste time. Yet much of what we have discussed since the last exam appears to be making a fuss over obvious details. Why be so careful? Why is David Hilbert making such a fuss over things like betweeness, and congruence, and the like. Is there a reason for this?

(5 pts. Extra credit): Do lines have ends? If not, why not? If so, how?

Axioms and major propositions

EP1: For every point P and for every point Q not equal to P there exists a unique line **l** that passes through P and Q

EP2: For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE.

EP3: For every point O and every point A not equal to O there exists a circle with center O and radius OA.

EP4: All right angles are congruent to each other.

Euclidean Parallel Postulate: For every line **l** and for every point P that does not lie on **l** there exists a unique line m through P that is parallel to **l**.

LR1: The following are the six types of justifications allowed for statements in proofs:
(1) By hypothesis (given)
(2) By axiom/postulate ...
(3) By theorem ... (previously proved)
(4) By definition ...
(5) By step ... (a previous step in the argument
(6) By rule ... of logic

LR2: To prove a statement H => C, assume the negation of statement C (RAA Hypothesis) and deduce an absurd statement using the hypothesis H if needed in your deduction.

LR3: The statement "not (not S)" means the same thing as "S".

LR4: The statement "not[H=>C]" means the same thing as "H and not C".

LR5: The statement "not[P and Q]" means the same thing as "not P or not Q".

LR6: The statement "not (forall(x) S(x))" means the same thing as "there exists(x) not(S(x))"

LR7: The statement "not (there exists(x) S(x))" means the same thing as "forall(x) not S(x)".

LR8: (modus ponens) If P => Q and P are steps in a proof, then Q is a justifiable step.

LR9: (a) []P => Q] & [Q => R]] => [P => R].
(b) [P & Q] => P, [P & Q] => Q.
(c) [~Q => ~p] <=> [p => Q]

LR10: For every statement P, "P or ~P" is a valid step in a proof.

LR11: Suppose the disjunction of statements S1 or S2 or ... or Sn is already a valid step in a proof. Suppose that proofs of C are carried out from each of the case assumptions S1, S2, ... Sn. Then C can be concluded as a valid step in the proof (proof by cases).

IA 1: For every point P and for every point Q not equal to P there exists a unique line l incident with P and Q.

IA 2: For every line l there exists at lease two points incident with l

IA 3: There exist three distinct points with the property that no line is incident with all three of them.

Prop 2.1: Non parallel distinct lines have a unique point in common
Prop 2.2: There exist three distinct non concurrent lines
Prop 2.3: For every line there is at least one point not lying on it.
Prop 2.4: For every point there is at least one line not passing through it.
Prop 2.5 : For every point P there exist at least two lines through P

BA 1: IF A*B*C then A, B, and C are three distinct colinear points, and C*B*A

BA 2: Given any two distinct points B and D, there exist points A, C, and E lying on the line through B and D such that A*B*D, B*C*D, and B*D*E.

BA 3: If A, B, and C are three distinct colinear points, then one and only one of the points is between the other two.

BA4: For every line l and for any three points A, B, and C not lying on l,

(i) if A and B are on the same side of l and B and C are on the same side of , then A and C are on the same side of l

(ii) If A and B are on opposite sides of l and B and C are on opposite sides of l, then A and C are on the same side of l

(corr:) If A and B are on opposite sides of l and B and C are on the same side of l, then A and C are on opposite sides of l