Math 280 B

FINAL EXAM

NAME____

General Notes:

1. **Show work**.
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam. Carry out any calculations to the point at which you would need a calculator (for example, to take the square root of a number) and leave it in that form.

Monday, Dec. 14, 2009
200 pts.

I. Topology and other basics

1. (5 pts. each) Give brief definitions of the following

   a. Open

   b. Boundary point

   c. Closed

   e. Simply connected

2. (5 pts.) Give a definition of what it means for a function *f(x,y,z)* to be continuous at a point (x_0,y_0,z_0)

4. (10 pts.) We know that the directional derivative $D_{\vec{u}} f$ for a differentiable function f and a unit vector **u** can be calculated as $\vec{\nabla} f \bullet \vec{u}$. Give the *definition* of $D_{\vec{u}} f$ at a point (x_0,y_0,z_0)

II. Dot and cross product.

1. Given $\vec{v} = 3\hat{i} + 2\hat{j} + \hat{k}$ and $\hat{w} = \hat{i} - 2\hat{j} + 3\hat{k}$

a. (5 pts.) Calculate $\vec{v} \bullet \vec{w}$

b. (10 pts.) Calculate $\vec{v} \times \vec{w}$

c. (10 pts.) What is the area of the parallelogram formed by the vectors **u** and **v** above?

II. Vector functions and their derivatives

1. Suppose that the vector function $\bar{r}(t) = 6t^3\hat{i} - 2t^3\hat{j} - 3t^3\hat{k}$ (problem 6, page 681).

a. (5 pts.) Compute $\bar{r}'(t) = \dfrac{d\vec{r}}{dt}$

c. (10 pts.) Find s(t), the length of the path from 0 to t. s(5), for example, should be the arc length of the path from t = 0 to t = 5

d. (5 pts.) Find the unit tangent vector **T(t)** of $\bar{r}(t)$

III. Partial derivatives

1. (10 pts.) Given $f(x, y, z) = x^2 + y^2 + z^2$ with $x(t) = 2\cos(t), y(t) = 2t, z(t) = 2\sin(t)$.

a. Write out the chain rule to calculate $\frac{df}{dt}$ in a general case (i.e, for some function f(x, y, z) and some curve $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$.

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b. Use the chain rule to calculate $\frac{df}{dt}$ in this case.

2. (10 pts.) Find a unit normal vector to the surface defined by $\frac{x^2}{4}+\frac{y^2}{4}+\frac{z^2}{9}=1$ at the point $(1,1,\frac{3}{\sqrt{2}}$.

3. (5 pts.) From your answer to problem 2 above, write the equation for the tangent plane to the surface at that point.

4. (5 pts.) The function $f(x,y) = \frac{x^2}{4} - \frac{y^2}{9}$ has a critical point at (0,0). Use the partial derivative test to discover whether this is a maximum, minimum, or saddle point.

IV. Integration

1. (10 pts.) Set up the integration necessary to calculate the volume of the region enclosed by the surface $\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{9} = 1$. You might find it easier to start with $-3 \le z \le 3$. Do **not** evaluate the integral.

2. Consider the region bounded by the points (0,0,0), (1, 0, 0), (0, 1, 0), and (0, 0, 1).

   a. (5 pts.) Sketch the region

   b. (15 pts.) Decide on how the region is to be described and calculate the volume of the region.

V. Integration in Vector Fields.

1. (15 pts.) Calculate the line integral $\int_C (x+y+z)ds$ where C is the straight line between (1,1,1) and (2, 3, 5)

2. (10 pts.) Calculate the work done against a force field $\vec{F}(x,y,z) = x\hat{i} - xy\vec{j} - y\hat{k}$ along the straight line between (1,1,1) and (2, 3, 5)

3. (10 pts.) The force field $\vec{F}(x, y, z) = x\hat{i} + y\vec{j} + z\hat{k}$ is conservative. Using the techniques of the last chapter, find the potential function (i.e., the scalar function whose gradient is the given function **F**). Note - you can probably simply write down the potential function (it is one we've seen before), but I want to see a clear exposition of the steps in the process.

4. (10 pts.) State both forms of Green's theorem with preconditions (i.e., what must be true for the theorem to hold).

5. (15 pts.) Verify Green's theorem in one of its two forms for the function $\vec{F}(x, y) = -y\hat{i} + x\hat{j}$ in the region bounded by the unit circle centered at the origin.(a modification of problem 1 on page 884). 10 pts. for verifying the flux-divergence form, full credit for verifying the circulation-curl form (remember trig substitution and the half-angle formulae)).