Math 210

Fourth Hour Exam

Name _____

Notes:

1. Show your work. Answers given without indication of how you got the answer may not receive credit.
2. Calculators are permitted on this exam. In problems involving calculating a number, however, it is sufficient to leave the calculation in a form in which it can be entered into a calculator (**except where a number is called for in the problem**). If you do use a calculator, you should include this "pre-calculator" expression as part of your answer.

Friday, April 29
100 pts.

I. Yet another mathematical induction problem. (15 pts.) **(corrected)**

Following the following steps, prove that $f_1 + f_3 + \ldots + f_{2n-1} = f_{2n}$ where f_n is the n^{th} Fibonacci number (with $f_1 = f_2 = 1$, and where $f_n = f_{n-1} + f_{n-2}$ for n>2). Part of your grade on this problem will be the careful statement of each step in the process.

a. Carefully state and prove the **base case**.

b. State the **inductive step**. What is the **inductive hypothesis** in this case?

c. Prove the inductive step, carefully showing where the inductive hypothesis is used.

II. Big-O and Big-Θ (5 pts. each)

a. Define Big-O (formal definition, as in "f is O(g)")

b. Define Big-Θ (formal definition, as in "f is Θ (g)"))

c. If I have a sorting algorithm whose cost function is O(n*log(n)) and one which is O(n^2) (where **n** is the number of entries to be sorted), which one am I likely to use for increasing **n**? Why?

III. Recurrence relations.

1. (5 pts.) Evaluate (to a number, as far as you can without a calculator)

$$\sum_{k=0}^{9}\left(\frac{1}{2}\right)^{k}$$

2. (10 pts.) Given the recurrence relation

$$T(n)=\begin{cases}4T(n-1)+3 & n>0\\ 2 & n=0\end{cases}$$

Write (but do not calculate) the closed form expression for T(10)

3. (10 pts. - **corrected**) Suppose we have

$$T(n)=\begin{cases}8T(\frac{n}{2})+n & n>1\\ 2 & n=1\end{cases}$$

What is the Big-Θ bounds on the closed-form solution for T(n)? Simplify your answer as much as possible.

IV (20 pts.) Compilation and formal languages

Give an overview of the compilation process as discussed in class, describing what happens at each step in the process. Identify where finite state automata are used in the process.

V Finite State Automata (FSA)

Consider the following state transition table.

input \ state	state 1	state 2	state 3	state 4
A	goto state 2	error	goto state 4	error
B	error	goto state 3	goto state 3	error

1. (10 pts.) Sketch the Finite State Automaton using the notation discussed in class. State 1 is the start state, and state 4 the only final state.

2. (10 pts.) Trace the finite state automaton (giving input string and state at each step) on the string ABBA. Is the string accepted?

(problem V continued)

c. (5 pts.) Try and give a characterization of the strings of characters with only A's and B's that are accepted by this FSA.