Math 210

Fourth Hour Exam

Name _____

No calculators should be necessary for this exam

(unless otherwise instructed, please leave your answer in a form which you could finish using only multiplication, division, addition, or subtraction (that is, using only a basic calculator without any additional functions)

Friday Dec. 5
100 pts

I. More counting

1. (15 pts.) Briefly describe each of the following two functions (that is, what do they count), give formulas for each, and calculate their values to a number (please complete the calculations for this problem).

P(6,3)

C(6,3) = $\binom{6}{3}$

2. (5 pts.) What is the coefficient of $x^3 y^7$ in the expansion of $(x+y)^{10}$?

3. (5 pts.) What is the coefficient of $x^4 y^6$ in the expansion of $(2x + y)^{10}$?

4. (10 pts.) Pascal's identity $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$ gives us a recursive approach to finding $\binom{n}{k}$. Using your favorite programming language, write a recursive procedure to calculate C(n, k). What are the base cases?

II. Some probability

1. Some probability (5 pts. each unless otherwise marked)

a. What is the probability of drawing a five-card hand containing exactly three kings?

b. What is the probability of throwing an even number or a 1 in a single throw of a fair die?

c. Suppose that a die has been fixed so that 4 shows up twice as often as the other numbers (1, 2, 3, 5, and 6). What is the probability of throwing a 4 in this case? Of throwing a 2?

d. Given the biased die in the previous problem, what is the probability of throwing an even number? An odd number?

e. Suppose that with this biased die, two players bet a dollar each with player A betting that an even number will appear and player B betting that an odd number will appear. Who stands to profit? By how much (on the average)

f. What is a Bernoulli trial?

g. Supposed a biased coin comes up Heads with probability (1/3) and Tails with probability (2/3). What is the probability that in 5 tosses of the coin Heads comes up exactly 3 times?

h. Define P(A|B)

i. State Bayes' theorem

j. (15 pts.) Suppose that we know the following: Be careful to show your work.

The probability of passing an exam is 70%
60% of students study for exams
The probability of passing an exam if you study is 90%
(What is the probability of not passing an exam if you study?)
The probability of passing an exam if you don't study is 30%

a. (10 pts.) What is the probability that a student studied given that the student passed the exam?

b. (5 pts.) What is the probability that a student did not study given that the student passed the exam?

6. (5 pts.) Say something appropriate (but non-obvious - statements like "Thomas Bayes gave us Bayes' rule" would likely not count for very much) about one of the following:

a) James Bernoulli
b) Pierre-Simon Laplace
c) Thomas Bayes
d) Blaise Pascal