

Math 210

First Hour Exam

Name _____

Please note: I will not be able to be with you for this exam. Professor Bryan Smith has kindly offered to proctor the exam. Please look the exam over, and begin with the questions you feel most comfortable with. Please recall that I do not intentionally ask trick questions (unless I clearly tell you that I am doing so). If something appears ambiguous or tricky, please do your best with it and add a note saying why the question appeared ambiguous. Many thanks, and good luck with the exam!

Friday, September 26
1:00 PM
Thompson 391
100 pts.

2. (15 pts.) Given the statement “If I win the lottery then I am rich” What is the

Sufficient condition?

Necessary condition?

Converse?

Contrapositive?

3. (10 pts.) Simplify the following expressions by moving the negation sign inside so that it appears only directly before predicate expression(s) $P(x)$ and $Q(x)$ and so that only **and**, **or**, and **not** are used in addition to the predicates and quantifiers (that is, translate implication statements using these three symbols). Also remove any double negations.

$$\neg \exists x (P(x) \wedge Q(x))$$

$$\neg (\forall x \exists y (P(x) \rightarrow Q(y)))$$

4. (10 pts.) Suppose that $R(x)$ means “ x is a resource”, $P(x)$ means “ x is a process”, $A(x)$ that “ x is available”, $N(x, y)$ for “ x requires y ” and finally that $L(x,y)$ means “ x locks y ” (as in “process x locks resource y ”. Express the following in English (translation, not transliteration: i.e., do not use words like “for all” and “there exists” and “it is not the case that”).

a. $\forall x \forall y ((P(x) \wedge R(y) \wedge N(x, y) \wedge A(y)) \rightarrow L(x, y))$

b. $\forall x \forall y (P(x) \wedge R(y) \wedge L(x, y) \rightarrow \neg A(y))$

5. (10 pts.) Complete the following truth table for the expression $((p \rightarrow q) \leftrightarrow (\neg p \vee q))$. Is the expression a tautology?

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \vee q$	$((p \rightarrow q) \leftrightarrow (\neg p \vee q))$

6. (5 pts. each). In each case below, say whether or not the conclusion follows from the statements above. If so, name the rule. If not, say why not
- If George is clever he will do well. George is clever. Therefore he will do well.
 - If George is clever he will do well. George did not do well. Therefore he is not clever.
 - George is bright or George is struggling. George is not struggling. Therefore George is bright.
 - If George is clever he will do well. George did well. Therefore he is clever.

7. (20 pts.) Suppose that set $A = \{b, d\}$, and that $B = \{a, b, c\}$. What do we get for

$$A \cup B$$

$$A \cap B$$

$$A - B$$

$$A \times B$$

8. (5 pts.) Say something relevant to the course about one of the following:
- George Boole
 - René Descartes
 - Augustus De Morgan
 - Georg Cantor
 - Ada Augusta, Countess of Lovelace