Math 210

Third Hour Exam

Name ______________________________

No calculators should be necessary for this exam

(unless otherwise instructed, please leave your answer in a form which you could finish using only multiplication, division, addition, or subtraction (that is, using only a basic calculator without any additional functions)

Friday Nov. 16
90 pts (will be normalized to
100 pts. in the gradebook)

1. (10 pts.) Suppose that

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \text{ and that } B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}. \text{ Calculate}$$

$A \wedge B$ (join)

$A \otimes B$ (Boolean product - it was the closest symbol I could find in Equation Editor)

2. Some counting questions (5 pts. each unless otherwise marked)

a. A part number is made up of two uppercase letters followed by three digits. How many part numbers can we construct in this way? What principle of counting are we using?

b (10 pts.) Suppose now that a part number can either be made up of two uppercase letters followed by three digits (as in part (a)), **or** of three uppercase letters followed by two digits. How many part numbers can we construct in this way? What principle of counting are we using?

c. (10 pts.) Give definitions of the following two functions, and calculate their values to a number (please complete the calculations for this problem).

P(5,3)

$C(5,3) = \binom{5}{3}$

d. What is the coefficient of $x^7 y^3$ in the expansion of $(x + y)^{10}$?

e. What is the coefficient of $x^7 y^3$ in the expansion of $(x + 2y)^{10}$?

f. What is a combinatorial proof?

3. (10 pts.) Show that for any positive integer $d > 1$ and for any collection of $(d+1)$ distinct numbers, at least two of those numbers have the same remainder when divided by d. What principle are you using?

4. (10 pts.) The identity $\binom{n+1}{k} = (n+1)\binom{n}{k-1}/k$ for $k \neq 0$ gives us a recursive approach to finding $\binom{n}{k}$. Using your favorite programming language, write a recursive procedure to calculate C(n, k). What are the base cases?

5. (15 pts.) let f_k be the kth Fibonacci number (remembering that $f_0 = 0, f_1 = 1, f_2 = 1, etc.$ Use mathematical induction to prove that $f_1^2 + f_2^2 + f_3^2 + \cdots f_n^2 = f_n f_{n+1}$

6. (5 pts.) Say something appropriate about one of the following:

a) James Bernoulli
b) Pierre-Simon Laplace
c) John McCarthy
d) Fibonacci (Leonardo of Pisa)
e) G. Lejeune Dirichlet
f) Blaise Pascal