Math 210

Second Hour Exam

Name ______________________________

No calculators should be necessary for this exam

Friday, October 26
100 points

1. (10 pts.) Use the rules for summation and the formula which tells us that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ to demonstrate that $\sum_{k=1}^{n} (2k-1) = n^2$

2. (10 pts.)

Give a formal definition of what it means that the function f is O(g).

Give a formal definition of what it means to say that a function f is _(g)

Give a formal definition of what it means that the function f is _(g).

3. (10 pts.) Find witnesses to demonstrate that $\frac{n(n+1)}{2}$ is $O(n^2)$. Show your work, and say (briefly) why the witnesses you selected work (i.e., it is not sufficient to simply write down some numbers - please give some explanation about why they work.

4. (10 pts.) There are a number of algorithms for multiplying square n-by-n matrices. The first one we usually see is O(n^3). Another one (which you may see in CSci 361) is O($n^{2.81}$). For large n, which might you select and why? What might prompt you to reconsider for small n? For large n? (essay question).

5. (15 pts.) Find the internal representation (two’s complement) of -100. Give your (16 bit) answer in hex. Please note that this requires you to (1) find the binary representation of 100, (2) form the two's complement representation of -100, and (3) convert the resulting bit string to hexadecimal.

5. (10 pts.) Calculate in base 2 (showing work as appropriate)

```
1010
+  11
------
```

```
1010
-  11
------
```

```
1010
x101 (product)
-----
```

$101\overline{)111101}$ (give quotient and remainder)

6. (5 pts. each)

a. What does it mean to say (i.e., what is the definition of) $a|b$?

b. What is the smallest positive integer n for which $n \equiv 17 \bmod 5$ is true?

7. (5 pts.) Briefly state the technique for proving a statement using mathematical induction

8 (15 pts.) Use mathematical induction to show that

$$\sum_{k=1}^{n}(2k-1)=n^2$$

Using the following steps:

a. State and prove the base case

b. State the inductive step as it applies to this problem.

(problem 8 continued)

c. What part of the inductive step (part b) is the inductive hypothesis (or assumption)?

d. Complete the proof by proving the inductive step.

8. (5 pts.) Say something (appropriate to the course) about one of the following:

Paul Gustav Heinrich Bachman
Karl Friedrich Gauss
Donald Knuth
Fibonacci
Marin Mersenne