Math 210

Second Hour Exam

Name ______________________________

No calculators should be necessary for this exam

Friday, October 20
100 points

1. (5 pts. each)

Give a formal definition of what it means that the function f is O(g).

Give a formal definition of what it means to say that a function f is _(g)

Use these definitions to demonstrate that f is O(g) if and only if g is _(f).

2. (10 pts.) Find witnesses to demonstrate that $2x^3 + 5x^2 - 12x + 3$ is $O(x^2)$. Show your work.

3. (10 pts.) Linear search is O(n) and binary search is O(log(n)). Say informally what this means (as if to a programmer who wants to make a decision between the two algorithms). As a part of your answer, argue that in some cases you might want to still pick a linear search over a binary search.

4. (15 pts.) Find the internal representation (two’s complement) of -125. Give your (16 bit) answer in hex. Please note that this requires you to (1) find the binary representation of 125, (2) form the two's complement representation of -125, and (3) convert the base two number that results to hexadecimal.

5. (20 pts.) Calculate (showing work as appropriate)

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1101
+  11
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1101
- 101
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1011
x101 (product)
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$101\overline{)101111}$ (give quotient and remainder)

6. (5 pts.) Briefly state the technique for proving a statement using mathematical induction

7 (15 pts.) Use mathematical induction to show that

$$\sum_{k=0}^{n} 2^n = 2^{n+1} - 1$$

Using the following steps:

a. State and prove the base case

(continued on the next page)

b. State the inductive step as it applies to this problem.

c. What is the inductive hypothesis (or assumption)?

d. Complete the proof by proving the inductive step.

8. (5 pts.) Say something (appropriate to the course) about one of the following:

Paul Gustav Heinrich Bachman
Karl Friedrich Gauss
Donald Knuth
Fibonacci