Math 211

Second Hour Exam

Name ______________________________

No calculators should be necessary for this exam

Friday, October 14
100 points

1. (10 pts.)

a. Give a formal definition of what it means that the function f is O(g).

b. Give an informal definition of what it means to say that a function f is _(g)

2. (10 pts.) Find witnesses to demonstrate that $4x^2 + 5x\ln(x) \square 127$ is $O(x^2)$. Show your work. Remember that ln(x) < x for all x > 1.

3. (5 pts.) Using limits, demonstrate that xln(x) is is $O(x^2)$.

4. (10 pts.) Using Euclid's algorithm, find the GCD of 84 and 105. Show your work.

5. (10 pts.) Find the internal representation (two's complement) of -125. Give your (32 bit) answer in hex.

6. (10 pts.) Calculate (showing work as appropriate)

```
1101
+110
------
```

```
1101
-111
------
```

```
1101
x110        (product)
-----
```

7. (5 pts.) Calculate the following arithmetic expressions mod 10. Express your answer using the smallest positive integer.

6 + 7

6*7

8. (15 pts.) Suppose that

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 3 & 0 \end{bmatrix} \text{ and that } B = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}. \text{ Calculate}$$

A + B

A*B (matrix product)

9. (5 pts.) Briefly state the technique for proving a statement using mathematical induction

10 (15 pts.) Use mathematical induction to show that

$$\sum_{k=1}^{n} k * (k!) = (n+1)! - 1$$

Using the following steps:

a. State and prove the base case

(continued on the next page)

b. State the inductive step as it applies to this problem.

c. What is the inductive hypothesis (or assumption)?

d. Complete the proof by proving the inductive step.

11. (5 pts.) Say something (appropriate to the course) about one of the following:

a. Marin Mersenne
b. Karl Friedrich Gauss
c. Euclid
d. Pierre de Fermat