## Math 160 K

## Fourth HOUR EXAM

NAME

## General Notes:

1. Show work. A correct answer without supporting work may not be given credit.
2. Look over the test first, and then begin.
3. Calculators are permitted on this exam, but only for basic arithmetic (i.e., no statistical calculations)

Friday, December. 4, 2009
100 pts.

## I. Binomial distributions

1. ( 10 pts.) What is the binomial setting? That is, what do we look for in deciding that the binomial distribution is the correct one to use in a given setting?
2. A fair die (all results equally likely) is tossed 12 times and the number of times a ' 5 ' appears is counted. ( 5 pts. each)
a. Explain why this is suitably described by a binomial distribution $B(x, y)$ and say what x and y are in this case.
b. How many times do we expect a " 5 " to turn up in those 12 trials? What is the formula for this?

## (Problem 2 continued from previous page)

b. What is the standard deviation of the number of five's in those 12 trials? What formula do you use to compute this?
c. Use Table $C$ to find the probability of observing a "five" 3 times or fewer in these 12 trials?
II. Sampling distributions and the Central Limit Theorem

1. (5 pts. each) We are interested in the high school GPA (HSGPA) of all students graduating from Washington high schools in a particular year. Suppose that the HSGPA follows a $\mathrm{N}(2,0.5)$ distribution (I'm making these numbers up - the actual average is close to 3.0). Suppose that we take an SRS of 100 of these students and find the average (mean) GPA for this sample.

Suppose now that we do this for all possible SRS's of 100 students, and look at the distribution of the average HSGPA from all of these groups.
a. Supposing that a population distribution follows a $\mathrm{N}(\mu, \sigma)$ distribution. In general, what distribution does the computed mean from all simple random samples of $n$ individuals follow?
b. What distribution does the above collection of sample mean HS GPA's follow? Please provide both mean and standard deviation.
2. (10 pts.) State the Central Limit Theorem, and give a reason why it is a surprising result.
III. Confidence intervals and margin of error

Suppose that we take an SRS of 100 individuals from a population following a $\mathrm{N}(\mu, 10)$ distribution with $\mu$ unknown and calculate a sample mean $\bar{x}=4$.

1. (5 pts.) What does it mean to say that, in this case, the sample mean is an unbiased estimator of the population average? Please be specific.
2. (10 pts.) Give a $95 \%$ confidence interval for the population mean and say what it means.

## (Problem III continued)

3. (5 pts.) Rephrase your answer to (2) above in terms of bounds on error.
4. (5 pts.) We would like to cut the bounds on error in half. If everything else is kept constant, what sample size should we use?
IV. Hypothesis testing (5 pts. each)

Suppose we have a population with unknown mean but with a standard deviation of 100 . Studies have indicated that the mean might be 300 , but we believe that the actual mean is greater than that. We take a SRS of 100 individuals and calculate the sample mean to be 330.

1. State the null and alternative hypotheses
2. Is this a one-sided or a two-sided test? Why?
3. Calculate the z statistic for this sample assuming the null hypothesis.
(problem IV continued)
4. Using table A, and assuming the null hypothesis, what is the likelihood (probability) of finding this or a more extreme sample?
5. Do we accept or reject the null hypothesis at the $\alpha=0.001$ level? At the $\alpha=0.005$ level?
