Math 122D

FINAL EXAM

NAME_____

Wednesday, Dec. 14

200 pts.

I. Theory

1. (10 pts.) Give a formal definition of $\int_a^b f(x)dx$, and then give an informal definition as you would use in explaining it to a fellow student.

2. (10 pts.) We have studied two theorems called “the fundamental theorem of calculus”. Give a formal statement of one, and explain why it might be considered surprising.

3. (5 pts.) Give a formal definition of $\lim_{n\to\infty} a_n = L$.

4. (5 pts.) Give a formal definition of $\sum_{n=1}^{\infty} a_n = L$

6. (10 pts.) What do we mean (formally) by saying that a series is a Taylor series expansion of the function f(x) about x=c?

II. **Techniques (10 points each):** Evaluate the following definite and indefinite integrals. Show how you got your answer.

1. $\int_0^1 (2x^2 + x + 1)^2 (4x + 1)dx$

2. $\int \tan^2(x)\sec^2(x)dx$

3. $\int_0^{\frac{\pi}{4}} \cos^2 dx$ (limits are 0 to $\frac{\pi}{4}$)

4. $\int \frac{dx}{(x+1)(x-2)}$

5. $\int \cos(x)e^{x}\,dx$

6. $\int \frac{dx}{1+x^{2}}$

8. (15 pts.) Use the trapezoid rule with n = 4 to approximate $\int_0^1 x^2\,dx$. Carry your answer to the point where only numbers remain. (i.e.: To the point at which you could really use a calculator).

III. Applications

1. (15 pts.) Calculate the volume of the solid formed by rotating the region between the curves y = x and y = x^2 for $0 \leq x \leq 1$ about the x-axis.

2. (10 pts) Calculate the arc length of the curve $y = x^{\frac{3}{2}}$ for $0 \le x \le 4$. Set up only (i.e., take the calculation to the point at which only an integration needs to be done). *5 pts. extra credit*: Finish the calculation.

3. (10 pts.) State the theorem of Pappus, and use it to find the volume of the region formed by rotating the circle of radius 1 centered about the point (2,0) about the y-axis.

4. (15 pts.) Solve the differential equation $\frac{dQ}{dt} = 3Q(t)$ subject to the condition that Q(t) = 5 when t = 0. How long does it take to reach Q(t) = 10?

IV Power series

1. (5 pts each). Say whether the following two infinite series converges or diverges, and say why

   a. $\sum_{k=1}^{\infty} \frac{(k+1)^2}{k^2}$

   b. $\sum_{k=2}^{\infty} \frac{1}{k \ln k}$

2. (10 pts.) Write down the first three terms of the Maclaurin series for $f(x)=e^{x^2}$.

3. (10 pts.) What is the convergence set and the radius of convergence of the series $\sum_{k=1}^{\infty} \frac{(3x)^k}{2^{k+1}}$?