

Math 122D

FOURTH HOUR EXAM

NAME _____

General Notes:

1. Show work.
2. Look over the test first, and then begin.
3. No calculators on this exam.

Friday, Dec. 2, 2005
100 pts.

I. Ordinary Differential Equations

1. (15 pts.) Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = x$ subject to the initial conditions that $y = 1/3$ when $x = 1$.

2. Definitions and statement of theorems (5 pts. each)

1. Define $\lim_{k \rightarrow \infty} a_k$ (a mathematical definition, not an informal one)

2. Define S_n for the infinite sum $\sum_{k=1}^{\infty} a_k$.

3. How do we define the infinite sum $\sum_{k=1}^{\infty} a_k$ and when does it exist?

4. State the bounded monotone convergence theorem.
5. Say under what circumstances $\sum_{k=0}^{\infty} r^k$ converges and say what kind of series it is.
6. Does $\sum_{k=1}^{\infty} \frac{1}{x^3}$ converge? What sort of series is this?

II. Convergence tests (15 pts. each). **For each of the following problem state what the test shows (i.e., the theorem about the test), including conditions for the convergence test to work (5 pts.),** and then use the indicated convergence test to test for convergence (or divergence). Be sure to say what the result of your test is (converges/diverges). You do not need to prove that the series meets the requirements for the test.

1. Use the direct comparison test to test $\sum_{k=1}^{\infty} \frac{1}{k^2 + 3}$ for convergence

2. Use the integral test to test $\sum_{k=1}^{\infty} \frac{1}{(3k + 1)^2}$ for convergence

III.

a. (5 pts.) State the ratio test

b. (10 pts. each) use the ratio test to test the following series for convergence, saying whether the test reports convergence, divergence, or does not supply an answer:

$$\sum_{k=1}^{\infty} \frac{5^{2k+1}}{(2k+1)!}$$

IV. Use any test you like (but say which one you are using) to test the series $\sum_{k=1}^{\infty} \frac{2k+7}{k^5}$ for convergence.