

Math 122C

FINAL EXAM

NAME \_\_\_\_\_

Monday, Dec. 12

200 pts.

## I. Theory

1. (10 pts.) Give a formal definition of  $\int_a^b f(x)dx$ , and then give an informal definition as you would use in explaining it to a fellow student.

2. (10 pts.) We have studied two theorems called “the fundamental theorem of calculus”. Give a formal statement of one, and explain why it might be considered surprising.

3. (5 pts.) Give a formal definition of  $\lim_{n \rightarrow \infty} a_n = L$ .

4. (5 pts.) Give a formal definition of  $\sum_{n=1}^{\infty} a_n = L$ .



6. (10 pts.) What do we mean (formally) by saying that a series is a Taylor series expansion of the function  $f(x)$  about  $x=c$ ?

II. **Techniques (10 points each):** Evaluate the following definite and indefinite integrals. Show how you got your answer.

1.  $\int_0^1 (3x^2 + 1)(x^3 + x)^2 dx$

2.  $\int \sin^2(x) \cos(x) dx$

3.  $\int \sin^2(x) \cos^2(x) dx$

$$4. \int_0^1 \frac{dx}{1+x^2}$$

$$5. \int x e^x dx$$

$$6. \int \frac{dx}{x^2}$$

8. (15 pts.) Use Simpson's rule with  $n = 4$  to approximate  $\int_0^1 x^2 dx$ . Carry your answer to the point where only numbers remain. (i.e.: To the point at which you could really use a calculator).



### III. Applications

1. (15 pts.) The cross section of a trough is a right triangle with top 5', depth 10', and length 10'. If it is filled with water ( $\rho = 62.4 \text{ lb/ft}^3$ ) how much work is involved in pumping the water out?

2. (10 pts) Calculate the x-coordinate of the [center of mass of the] region bounded above by  $y = x$  and below by  $y = x^2$  for  $0 \leq x \leq 1$ .

3. (10 pts.) The force exerted by a spring is  $F = -3x$  where  $x$  is the extension from some rest point  $x=0$  inches. How much work is done by stretching the spring from its rest position ( $x = 0$ ) to  $x = 1$ ?

4. (15 pts.) Solve the differential equation  $\frac{dQ}{dt} = 3(10 - Q)$  subject to the condition that  $Q = 9$  when  $t = 0$ .

IV Power series

1. (5 pts each). Say whether the following two infinite series converges or diverges, and say why

a.  $\sum_{k=2}^{\infty} \frac{1}{k \ln k}$

b.  $\sum_{k=1}^{\infty} \frac{k+1}{(k+2)^2}$

2. (10 pts.) Write down the first three terms of the Maclaurin series for  $f(x) = e^{\sin(x)}$ .

3. (10 pts.) What is the [convergence set and] radius of convergence of the series  $\sum_{k=1}^{\infty} \frac{k^2 x^k}{k+1}$ ?