# Math 180 E

# FINAL EXAM

NAME____________________________________________________________

General Notes:

1. **Show work**.
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam.

Friday, December 14, 2012
200 points

## I. Limits and continuity

1. (5 points) Define formally what we mean when we say that $\lim_{x \to a} f(x) = L$.

2. (10 points) Given the function $f(x) = 3x + 7$, we want to show that $\lim_{x \to 2} f(x) = 13$. Considering your definition above, find a δ>0 which works for $\varepsilon = \frac{1}{100}$ and show that this works

3. (5 points) Define formally what we mean when we say that a function f(x) is **continuous** at a point **a**.

4. (10 points) Continuity has several important consequences. One of them is the Intermediate Value Theorem.

a. State the Intermediate Value Theorem

b. The function $f(x) = x^3 - x^2 + x - 1$ has a solution to $f(x) = 0$ somewhere in the interval [0, 2]. Use the Intermediate Value Theorem to say why this must be true.

5. (5 points). Another important consequence of continuity is the Extreme Value Theorem. Please state the theorem (as always, with preconditions).

6. (5 points each). Calculate the following limits.

a. $\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$

b. $\lim_{x \to 0} \frac{\sin(2x)}{x}$

c. $\lim_{x \to \infty} \frac{\ln(x)}{x}$ (hint: use l'Hôpital's rule)

## II. The Derivative

1. (5 points). Give a formal definition of the **derivative** of a function f(x).

2. (10 points) **Use the definition** to find the derivative of $f(x) = x^2 + 3x$

3. (5 points each) Find the derivatives of the following functions (with respect to **x**):

a. $f(x) = x^5 - 5x^2 + 7x + 47$

b. $f(x) = e^x \sin(x)$

c. $f(x) = \dfrac{e^x}{\sin(x)}$

d. $f(x) = e^{\sin(x)}$

e. $f(x) = \int_1^x \frac{1}{t} dt$

4. (5 points each)

a, What is a **critical point** of a function f(x)?

b. What are the critical point(s) (if any) of $f(x) = x^2 - 2x + 3$?

c. Use the second derivative test to determine if the critical point is a local maximum or minimum, saying why.

5. (15 points) A related rates problem (modified from Example 3 on page 197)
Water pours into a conical tank of height 10 feet and radius 5 feet at the rate of 6 cubic feet / minute. At what rate is the water rising when the water level is 5 feet? The volume of a right circular cone (the conical tank in this question) is given by $V = \frac{1}{3}\pi r^2 h$, where $r$ is the radius of the base of the cone and $h$ is the cone's height.

6. (15 points) (From Strauss, Bradley, and Smith Calculus) Suppose that it costs us $C(x) = \frac{1}{8}x^2 + 4x + 200$ dollars to manufacture and distribute **x** units of some commodity, and that we can sell each one for a price of (49-x) dollars per unit for a total revenue R(x) = $x(49 - x)$ dollars for x units. Our profit is then P(x) = R(x) - C(x). For what value of x will we obtain the largest profit?

7. (15 points) Find the equation of the line tangent to the curve given by $x^2 + y^2 - 4x + 2y = 20$ at the point (-1, 3)

III Integration

1. (5 pts.) What is a **partition** of an interval? Define and give an example of a partition of the interval [0,2] with three intervals.

2. (5 pts.) What is the **norm** of a partition (of an interval)? Define and say what the norm of the partition you gave in problem 1 is.

3 (10 points) What is a Riemann sum? Define, with an explanation of the individual parts in your definition.

5. (10 pts.) Define formally $\int_a^b f(x)dx$

6. (5 pts. each) A collection of integration problems, involving both definite and indefinite integrals. Where a definite integral is called for, please evaluate the definite integral to a number (you should be able to do this without a calculator). Remember to show your work.

a. $\int_0^1 (x^5 - 5x^2 + 7x + 47)dx$

b. $\int \cos(x)dx$

c. $\int_0^{\frac{\pi}{2}} \cos(x)dx$

d. $\int_0^1 \frac{1}{x^2+1}dx$