Math 180

THIRD HOUR EXAM

NAME____

General Notes:

1. **Show work**.
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam. Carry out any calculations to the point at which you would need a calculator (for example, to take the square root of the logarithm of a number) and leave it in that form.

Friday, April 9, 2010
100 pts.

I. Inverses and hyperbolics (5 pts each)

a. Find $ArcSin(\frac{-1}{\sqrt{2}})$ (inverse sin)

b. $\frac{d}{dx}ArcTan(x) =$

c. Define sinh(x)

d. $\frac{d}{dx}\cosh(x) =$

II. Approximations and rates of change

a. (5 pts.) What is the standard linear approximation to the function $f(x) = \sqrt{1+x}$ at the point x = 0? Use it to approximate the square root of 1.01. (problems 16, 17, page 218)

b. (15 pts.) A water reservoir has the shape of a right circular cone with a base radius of 9 feet and a depth of 18 feet. Water is being pumped into the cone at the rate of 10 cubic feet / hour. How fast is the water rising when the depth is 9 feet? Recall that the volume of a right circular cone is given by $V = \frac{\pi}{3} r^2 h$. Hint: use similar triangles.

III. Mean Value Theorem and some applications.

a. (5 pts.) State the mean value theorem (with preconditions)

b. (5 pts.) Verify the mean value theorem for the function $y = x^3$ on the interval [1, 2] (i.e., find a number c in the interval (1, 2) which works for the mean value theorem.

c. Consider the function $y = x^4 - 2x^2 + 1$

i. (15 pts) Find the critical points of the function and say where (over what intervals) the function is increasing and where it is decreasing.

ii. (10 pts.) Take the second derivative and classify the critical points as local maxima, local minima, and points of inflection, saying why in each case.

d. (10 pts.) Suppose that we know that the derivative of some function *f(x)* satisfies $\frac{df}{dx} = 3x^2 + 2$ and that f(1) = 5. What is f(x)?

IV Implicit differentiation

a. (15 pts.) Find the equation of the lines tangent and normal to the curve $x^2 + y^2 = 4$ at the point $(1, \sqrt{3})$ by first finding the slope at that point $\frac{dy}{dx}$ using implicit differentiation and then using the slope and the point to find the tangent and normal lines at that point.