

Math 180 F

THIRD HOUR EXAM

NAME _____

General Notes:

1. **Show work.**
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam. Carry out any calculations to the point at which you would need a calculator (for example, to take the square root of the logarithm of a number) and leave it in that form.

Friday, Nov. 13, 2009
100 pts.

I. Inverses and hyperbolics

a. (5 pts.) Find $\text{ArcCos}\left(\frac{1}{\sqrt{2}}\right)$ (inverse cos)

b. (5 pts.) $\frac{d}{dx}\text{ArcTan}(x) =$

c. (5 pts.) $\frac{d}{dx}\cosh(x) =$

II. Approximations and rates of change

- a. (10 pts.) Find the standard linear approximation to the function $f(x) = x^{\frac{1}{3}}$ at the point $x = 27$, and use it to approximate the cube root of 27.01.
- b. (10 pts.) The radius of a spherical cloud in space is expanding at the rate of 100 ft/sec. How fast is the volume of the cloud increasing when the radius is 1000 feet? The volume of a sphere is given by $V = \frac{4}{3}\pi R^3$

III. Mean Value Theorem and some applications.

a. (5 pts.) State the mean value theorem (with preconditions)

b. (5 pts.) Verify the mean value theorem for the function $y = x^2$ on the interval $[0, 1]$

- c. Consider the function $y = x(6 - 2x)^2 = 36x - 24x^2 + 4x^3$ (problem 12, page 268)
- i. (15 pts) Find the critical points of the function and say where (over what intervals) the function is increasing and where it is decreasing.
- ii. (15 pts.) Take the second derivative and classify the critical points as local maxima, local minima, and points of inflection, saying why in each case.

- d. (10 pts.) Suppose that we know that the derivative of some function $f(x)$ satisfies $\frac{df}{dx} = 2x + 2$ and that $f(1) = 4$. What is $f(x)$?

IV Implicit differentiation

- a. (15 pts.) Find the equation of the line tangent to the curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$ at the point $(\sqrt{3}, 2\sqrt{\frac{2}{3}})$ by first finding the slope at that point $\frac{dy}{dx}$ using implicit differentiation and then using the slope and the point to find the tangent line at that point.