Mathematics 434 **Spring 2014** **Problem Sheet 4**

Due March 28

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

"*The shortest path between two truths in the real domain passes through the complex domain.*" – Jacques Hadamard

"*Such is the advantage of a well-constructed language that its simplified notation often becomes the source of profound theories.*" – P. S. Laplace

"The road to wisdom? Well it plain and simple to express: Err and err and err again, but less and less and less." -Piet Hein, poet and scientist (1905-1996)

Problems

1. Determine the maximal ideals of $\mathbf{R}[x] / (x^2 - 3x + 2)$ where $\mathbf{R}$ denotes the real numbers.

2. Prove either of the following:

   (a) $\mathbf{Z}_2[x] / (x^3 + x + 1)$ is a field.

   (b) $\mathbf{Z}_3[x] / (x^3 + x + 1)$ is not a field.

3. Adapt Euclid's proof of the infinitude of prime integers to show that for any field F, there are infinitely many monic irreducible polynomials in $F[x]$.

   (a) Also explain why this argument fails for the formal power series ring $F[[x]]$.

4. Partial Fractions for polynomials

   (a) Prove that every rational function in $\mathbf{C}[x]$ can be written as a sum of a polynomial and a linear combination of functions of the form $1/(x-a)^i$.

   (b) Find a basis for $\mathbf{C}(x)$ as a vector space over $\mathbf{C}$.

5. Let a and b be relatively prime integers. Prove there are integers m, n such that $a^m + b^n \equiv 1 \pmod{ab}$