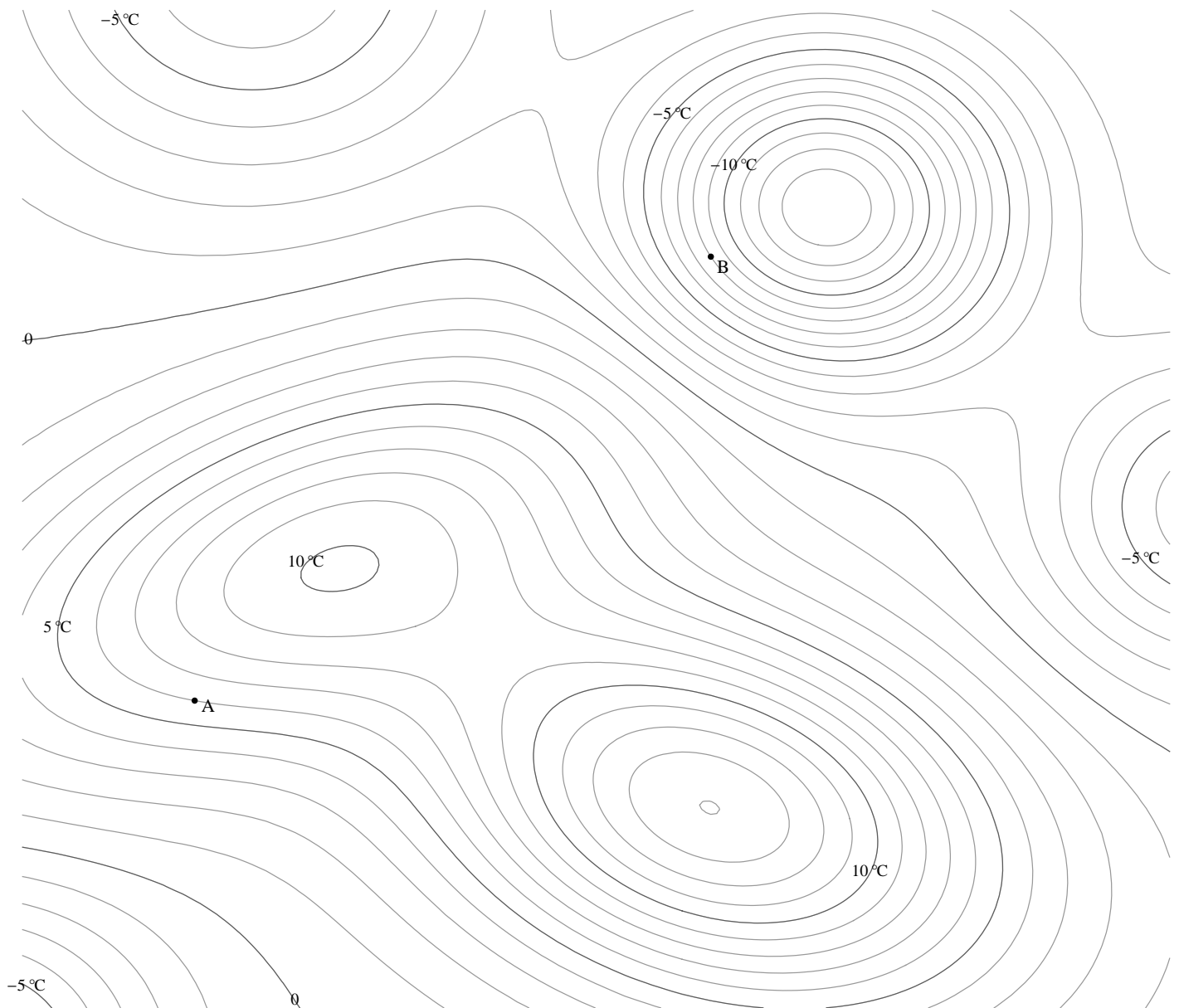


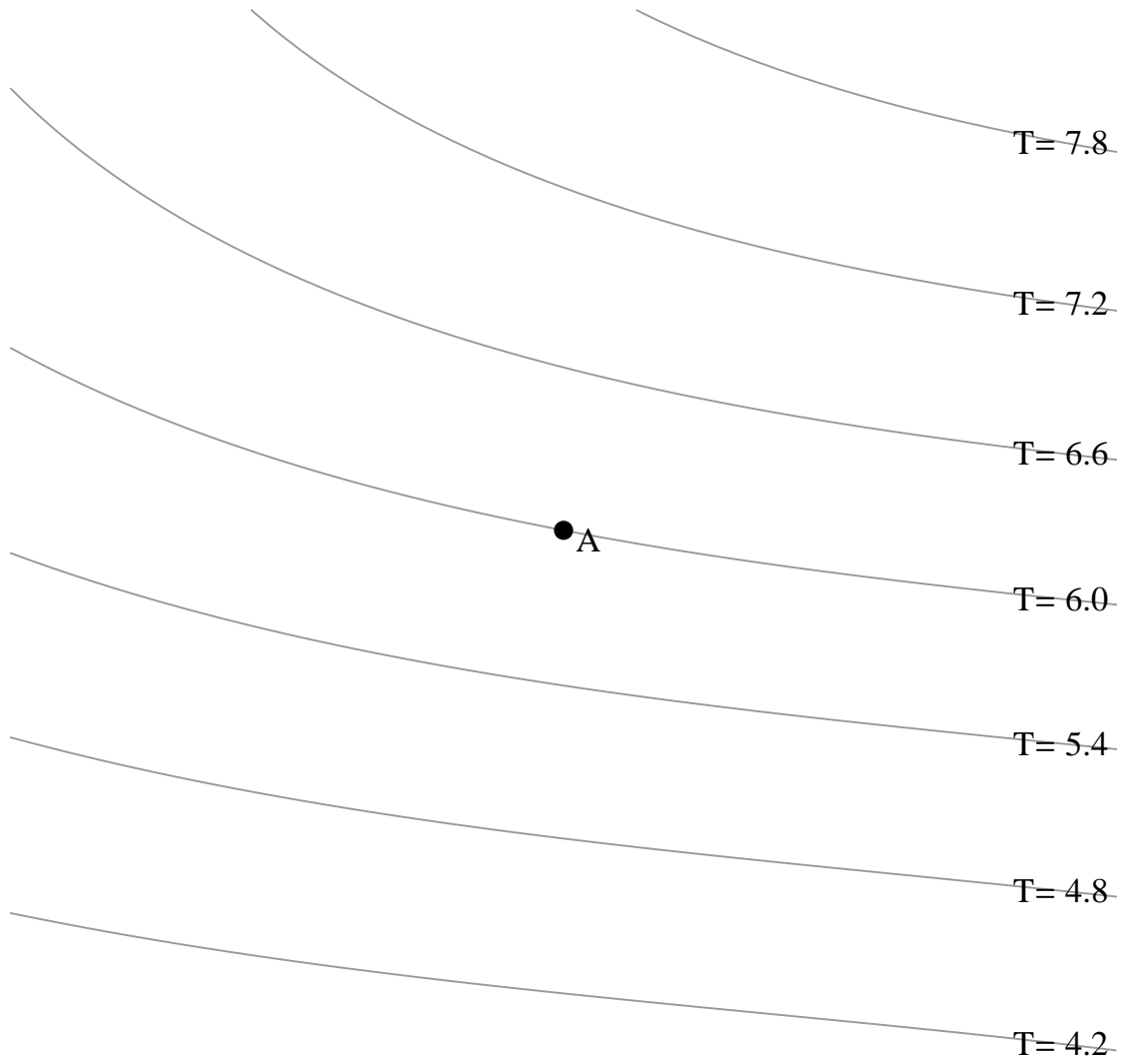
Estimating greatest rate of change

The accompanying plot shows level curves for a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$. We can think of each input as a point on a plane and the corresponding output as a temperature. We will consider distance to be measured in kilometers (km) and temperature to be measured in degrees Celsius ($^{\circ}\text{C}$). There is a scale for distance at the bottom of the plot. A selection of level curves is labeled with the corresponding temperature.

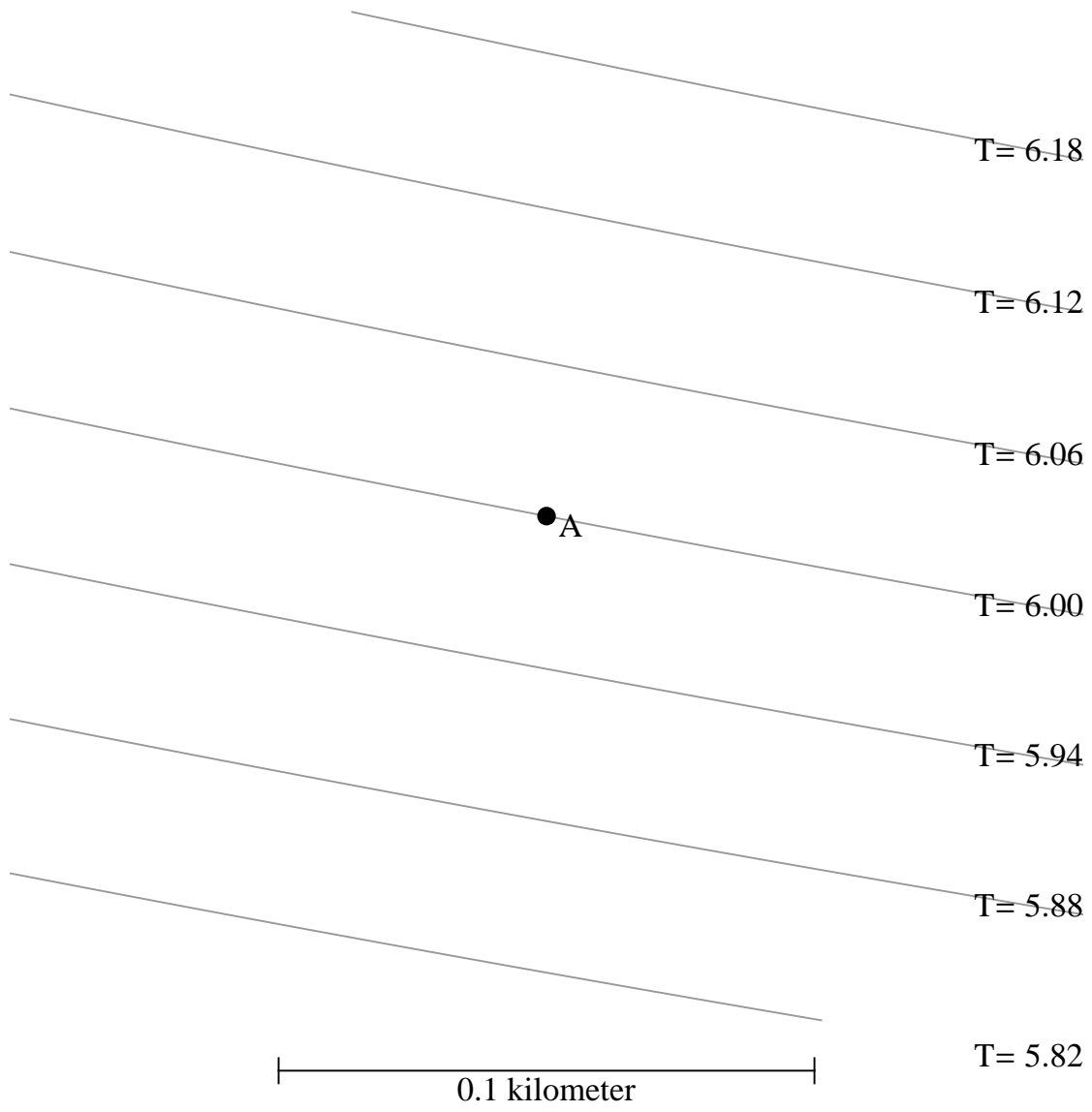
1. For the point A , estimate the direction of the greatest rate of change in temperature with respect to change in position.
2. For the point A , estimate the magnitude of this greatest rate of change.
3. Choose a scale for rate of change. Note that this scale is independent of the scale for distance. With the temperature interpretation, rate of change has units of degrees Celsius per kilometer ($^{\circ}\text{C}/\text{km}$) while the length scale is in kilometers (km). To choose a scale for rate of change, go to the bottom of the plot next to the given length scale and draw a horizontal vector (of any size you want) to represent a magnitude of $1^{\circ}\text{C}/\text{km}$. You will use this choice in what follows.
4. At the point A , draw a vector in the direction of the greatest rate of change having magnitude equal to that rate of change. Use the rate of change scale you chose in #3.
5. For the point B , estimate the direction of the greatest rate of change in temperature with respect to changes in position.
6. For the point B , estimate the magnitude of this greatest rate of change.
7. At the point B , draw a vector in the direction of the greatest rate of change having magnitude equal to that rate of change. Use the rate of change scale you chose in #3.



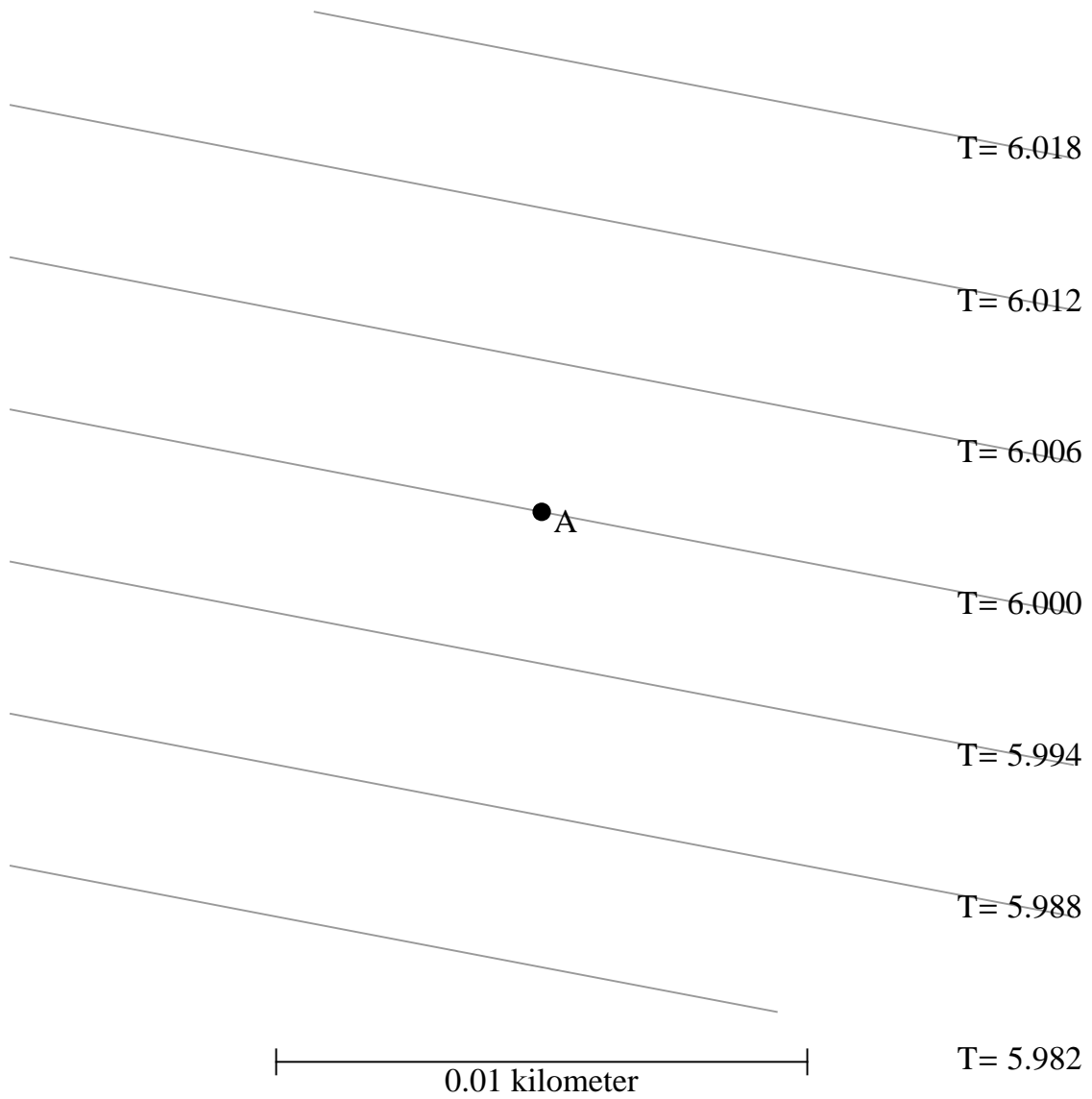
Level curves for temperature as a function of position.



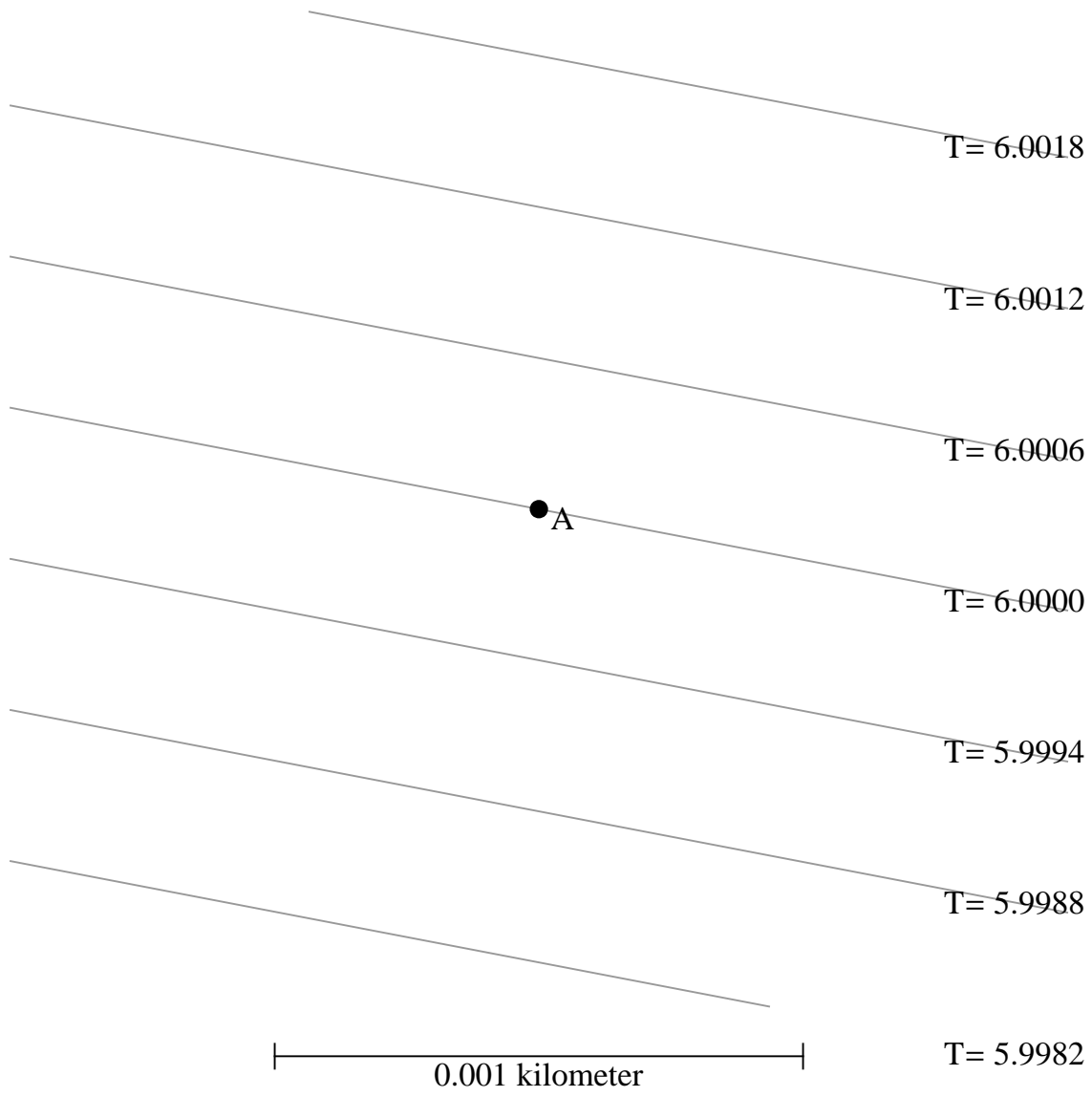
Zooming in on Point *A*



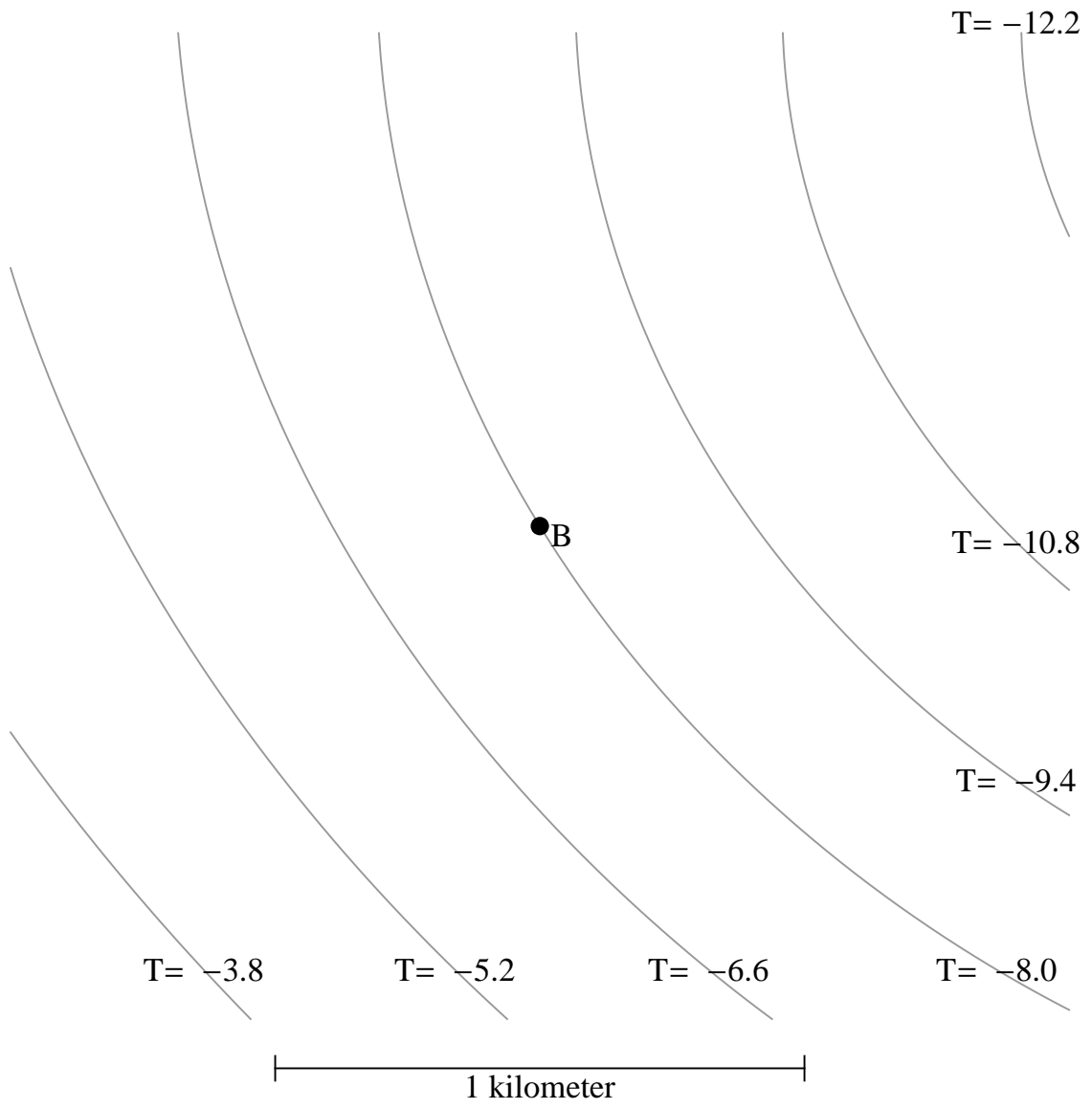
Zooming in on Point *A*



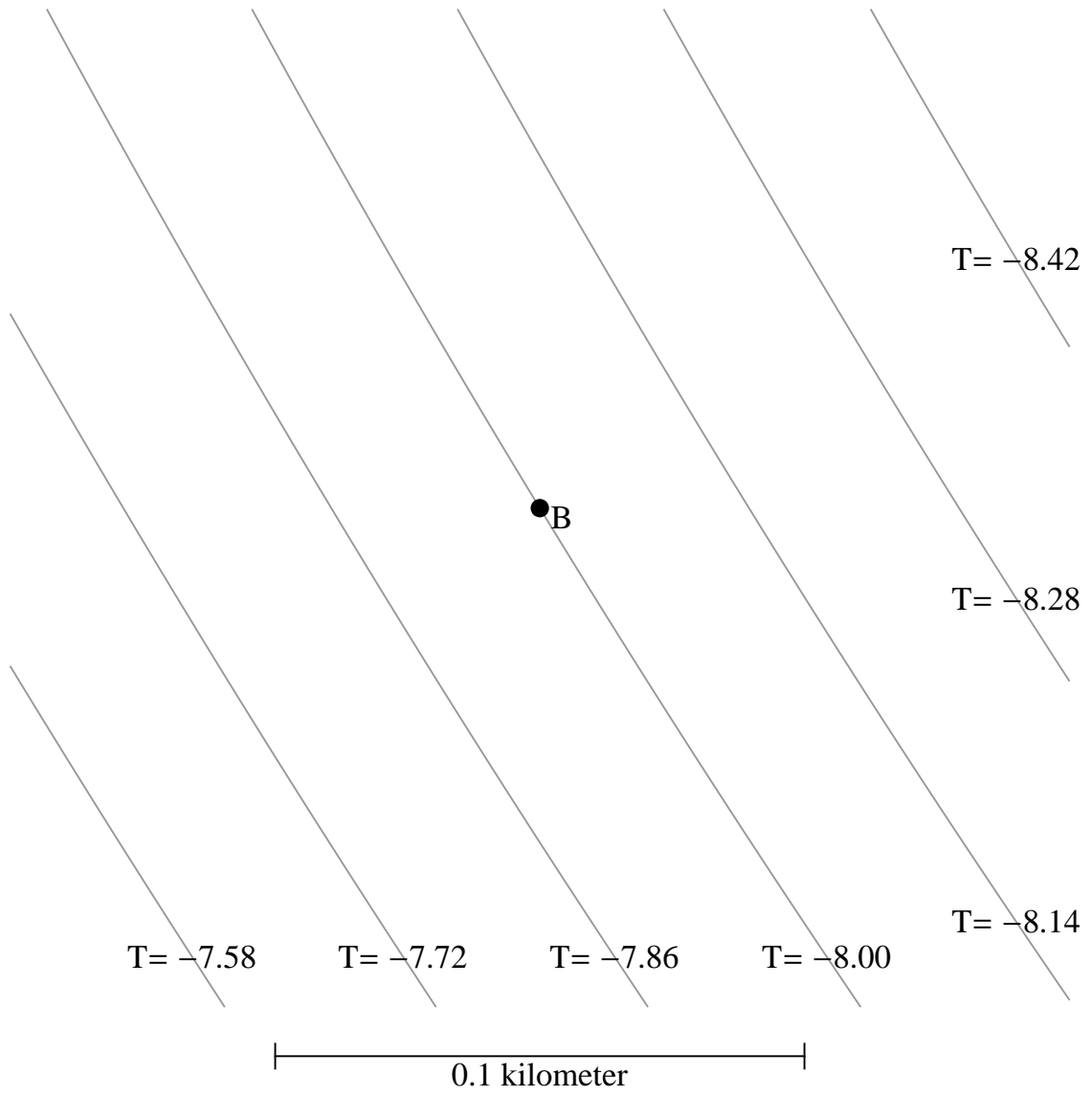
Zooming in on Point *A*



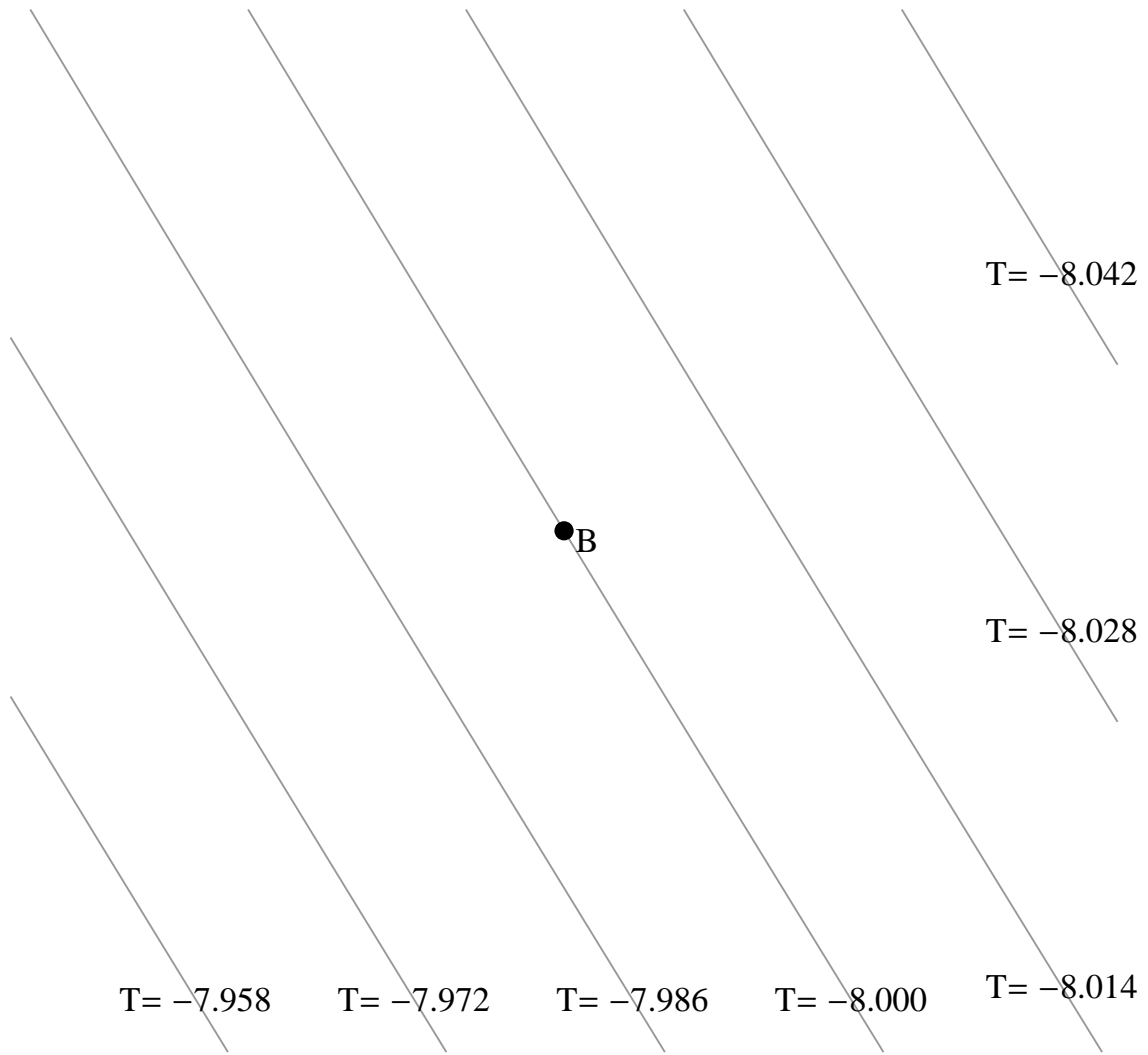
Zooming in on Point *A*



Zooming in on Point *B*

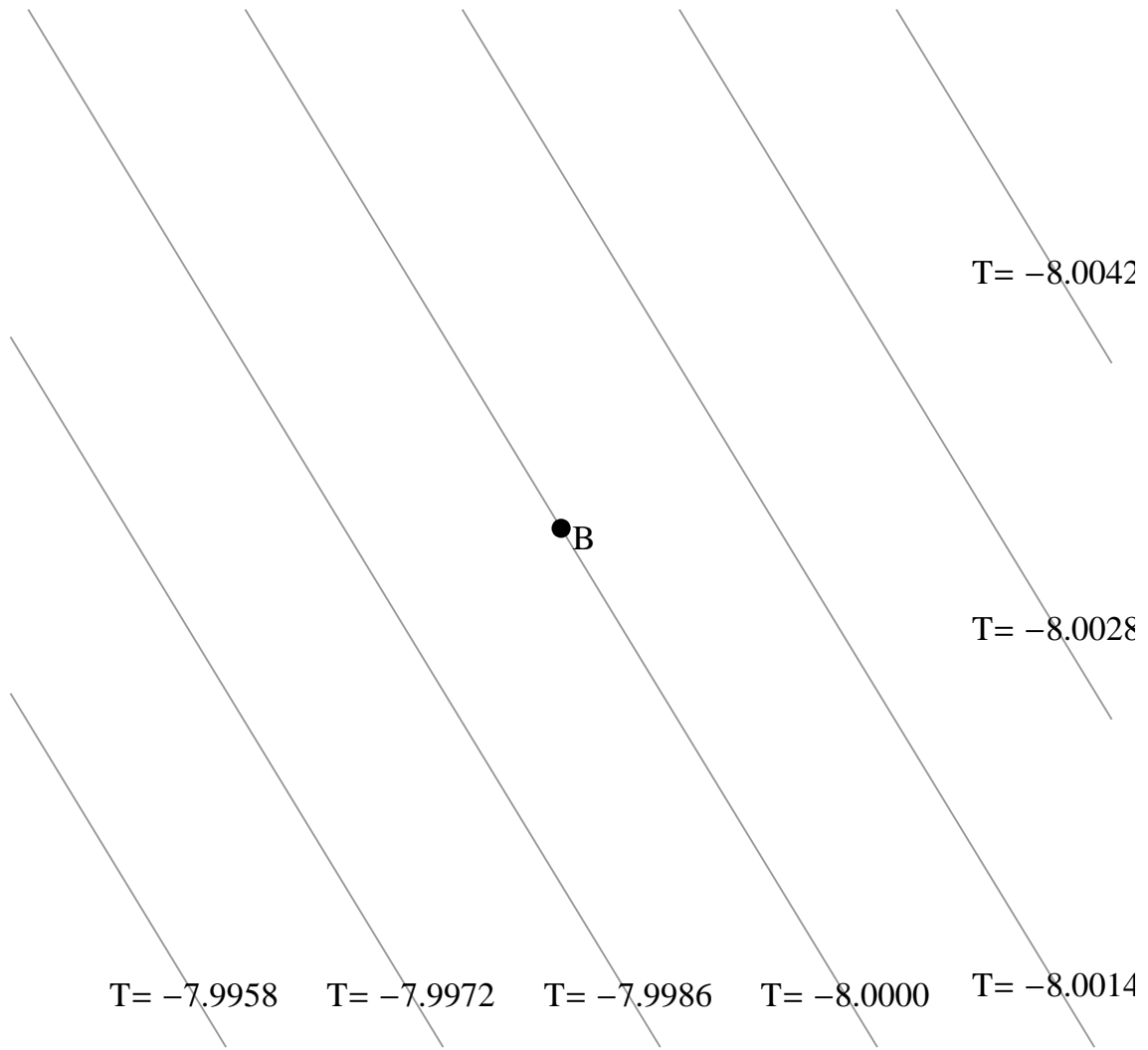


Zooming in on Point *B*



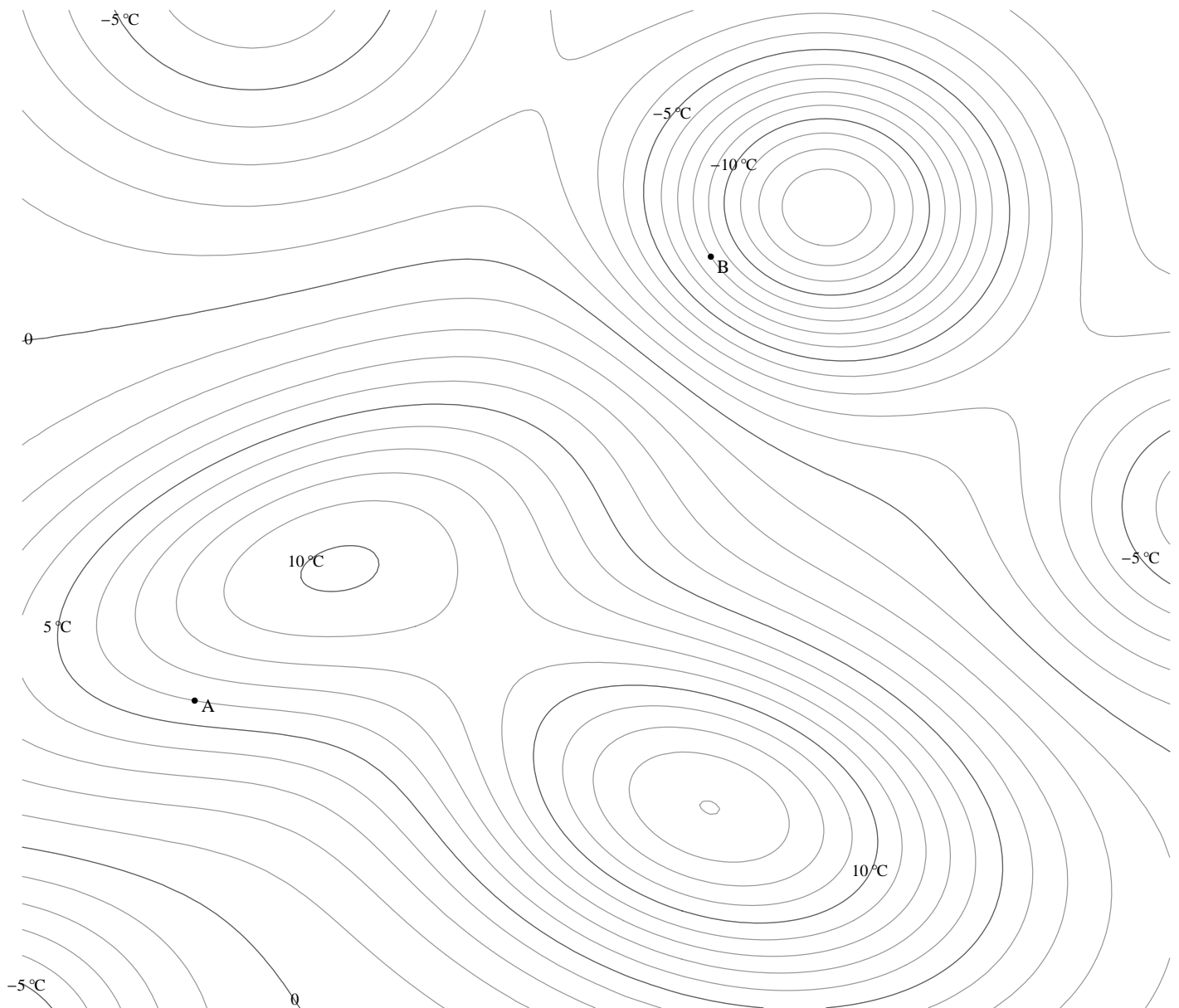
0.01 kilometer

Zooming in on Point *B*



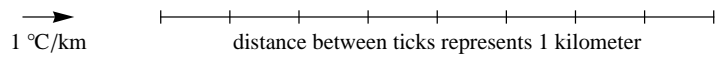
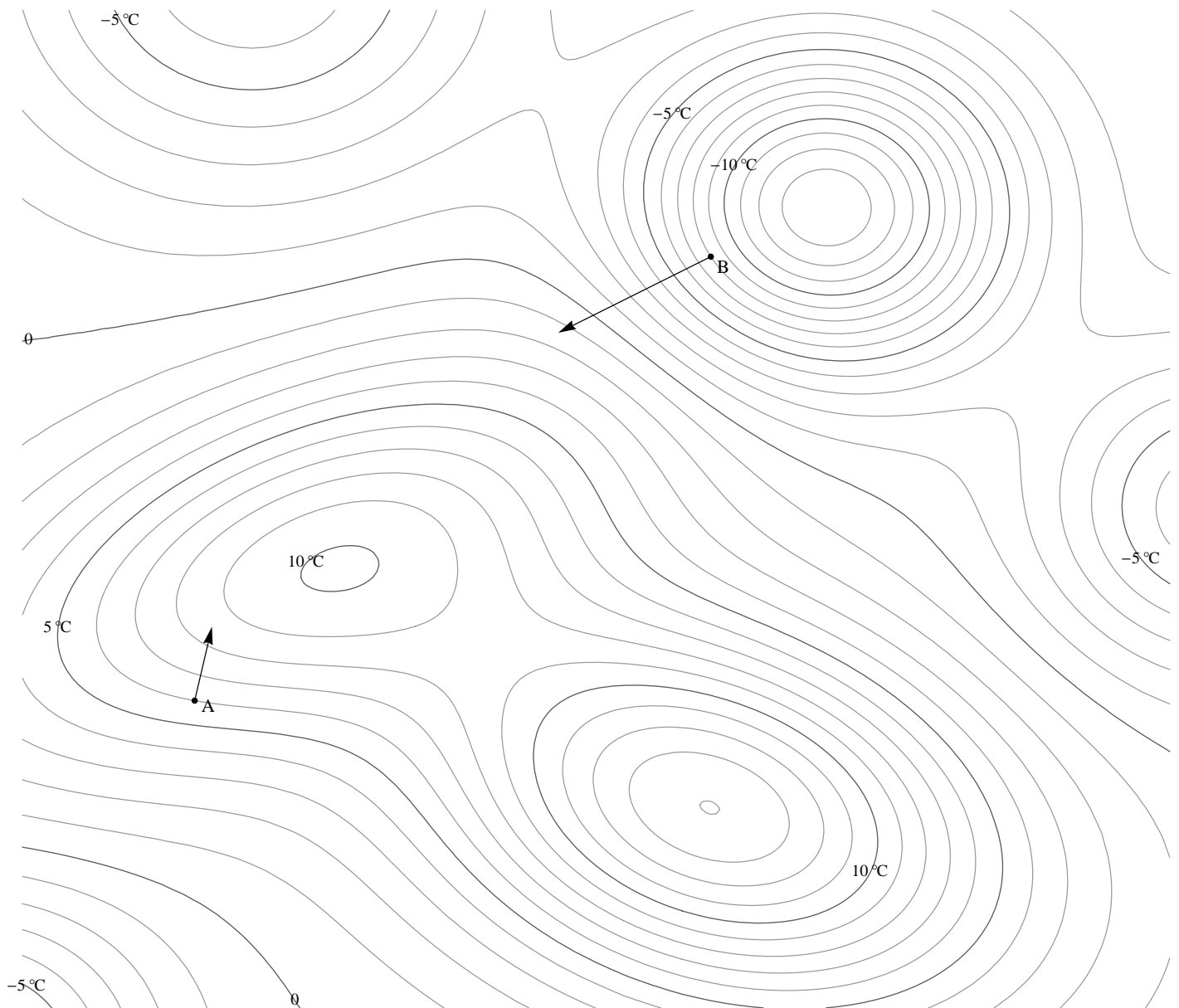
0.001 kilometer

Zooming in on Point B

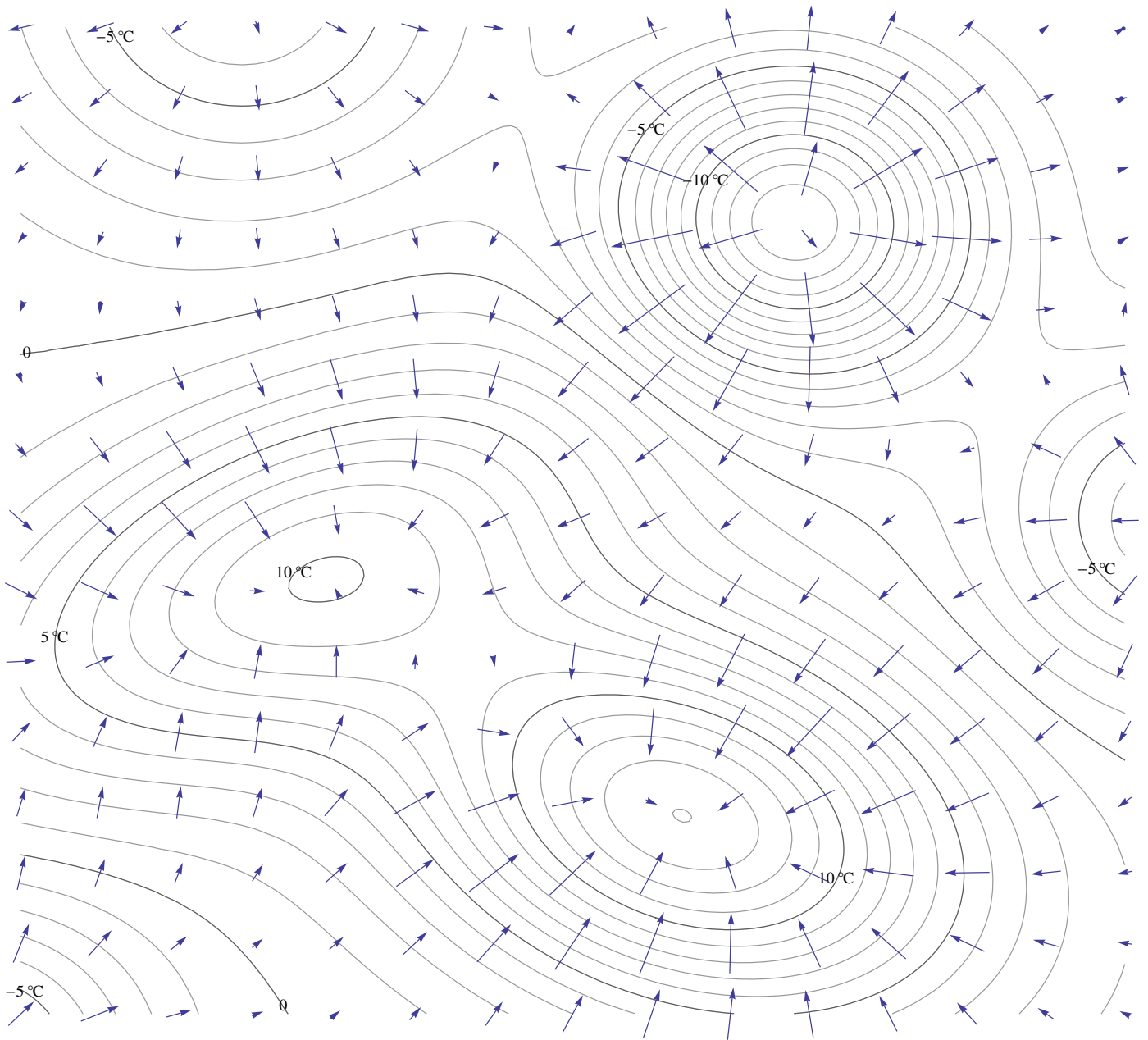


distance between ticks represents 1 kilometer

Level curves for temperature as a function of position.



Level curves with the vectors of Steps #4 and #7 included.
 Note that a scale for rate of change is also included.



Greatest rate of change vectors for a variety of points.
Note that the scale for rate of change in this plot differs from the scale in the previous plot.