Problems: Total from volume density

1. Relative to a chosen cartesian coordinate system, a solid object sits in the first octant bounded by \( z = 4 - x^2 - y \) and the coordinate planes. The object has a non-uniform composition so that the volume mass density is given by \( \delta(x, y, z) = 3z \). Compute the total mass of the solid.

   Answer: \( M = \frac{1024}{35} \)

2. A solid (right circular) cylinder of radius \( R \) and height \( H \) has a non-uniform composition so that the volume mass density is proportional to the distance from the lateral surface reaching a maximum \( \delta_0 \) along the central axis. Compute the total mass \( M \).

   Answer: \( M = \frac{1}{3} \pi R^2 H \delta_0 \)

3. Charge is distributed throughout a solid (right circular) cone of radius \( R \) and height \( H \) so that the volume charge density is proportional to the square of the distance from the vertex of the cone reaching a maximum of \( \delta_0 \) along the edge of the base of the cone. Compute the total charge \( Q \).

   Answer: \( Q = \frac{1}{10} \pi R^2 H \frac{R^2 + 2H^2}{R^2 + H^2} \delta_0 \)

4. A solid sphere of radius \( R \) has a non-uniform composition so that the volume mass density is proportional to the distance from the center of the sphere reaching a maximum of \( \delta_0 \) along the surface. Compute the total mass \( M \). Compare this mass to the total mass for a solid sphere of the same radius having uniform composition with mass density \( \delta_0 \).

   Answer: \( M = \pi R^3 \delta_0 \)

5. A solid sphere of radius \( R \) has a non-uniform composition so that the volume mass density is proportional to the distance from the surface of the sphere reaching a maximum of \( \delta_0 \) at the center. Compute the total mass \( M \). Compare this total mass to the total mass for a solid sphere of the same radius having uniform composition with mass density \( \delta_0 \). Also, compare this total mass to the total mass for the sphere in Problem 5.