## Estimating greatest rate of change

The accompanying plot shows level curves for a function $f: \mathbb{R}^{2} \rightarrow \mathbb{R}$. We can think of each input as a point on a plane and the corresponding output as a temperature. We will consider distance to be measured in kilometers (km) and temperature to be measured in degrees Celsius $\left({ }^{\circ} \mathrm{C}\right)$. There is a scale for distance at the bottom of the plot. A selection of level curves is labeled with the corresponding temperature.

1. For the point $A$, estimate the direction of the greatest rate of change in temperature with respect to change in position.
2. For the point $A$, estimate the magnitude of this greatest rate of change.
3. Choose a scale for rate of change. Note that this scale is independent of the scale for distance. With the temperature interpretation, rate of change has units of degrees Celsius per kilometer $\left({ }^{\circ} \mathrm{C} / \mathrm{km}\right)$ while the length scale is in kilometers (km). To choose a scale for rate of change, go to the bottom of the plot next to the given length scale and draw a horizontal vector (of any size you want) to represent a magnitude of $1{ }^{\circ} \mathrm{C} / \mathrm{km}$. You will use this choice in what follows.
4. At the point $A$, draw a vector in the direction of the greatest rate of change having magnitude equal to that rate of change. Use the rate of change scale you chose in \#3.
5. For the point $B$, estimate the direction of the greatest rate of change in temperature with respect to changes in position.
6. For the point $B$, estimate the magnitude of this greatest rate of change.
7. At the point $B$, draw a vector in the direction of the greatest rate of change having magnitude equal to that rate of change. Use the rate of change scale you chose in \#3.


Level curves for temperature as a function of position.


