Proof VS-1

Accepted **Not Accepted**

I affirm this work abides by the university's Academic Honesty Policy.

Print Name, then Sign

- First due date **Thursday, April 4**
- Turn in your work on a separate sheet of paper with this page stapled in front.
- Do not include scratch work in your submission.
- There is to be **no collaboration** on any aspect of developing and presenting your proof. Your only resources are: you, the course textbook, me, and pertinent discussions that occur **during class.**
- Follow the Writing Guidelines of the Grading Rubric in the course information sheet.
- Retry: Only use material from the relevant section of the text or earlier.
- Retry: Start over using a new sheet of paper.
- Retry: Restaple with new attempts first and this page on top.

"*By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems, and, in effect, increases the mental power of the race.*" – Alfred North Whitehead

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VS-1 (Section PD)

Definition 1 *Let $W_1, W_2, \cdots$ be any collection of subsets of a set V. Define* $\bigcap_{k=1}^{1} W_k = W_1$, *and* $\bigcap_{k=1}^{m+1} W_k = \left(\bigcap_{k=1}^{m} W_k\right) \cap (W_{m+1})$ *for all integers* $m \geq 1$.

1. Use the Principle of Mathematical Induction to prove the following theorem.

   Theorem 1 *If $W_1, W_2, \cdots, W_p$ are subspaces of a vector space V, then their intersection* $\bigcap_{k=1}^{p} W_k$ *is also a subspace of V.*

2. Show that no analogous theorem can be true for unions by specifying two particular subspaces of $\mathbf{C}^3$ whose union is not a subspace of $\mathbf{C}^3$. Be sure to explain why the union is not a subspace.
3. Use the concept of dimension to determine all subspaces of $\mathbf{C}^3$. Then describe the geometric meaning of each type of subspace for vectors that have real numbers as entries.

Notes:

- The intersection of sets S and T is defined by $S \cap T = \{x : x \in S \text{ and } x \in T\}$.
- The union of sets S and T is defined by $S \cup T = \{x : x \in S \text{ or } x \in T \text{ (or both)}\}$
