Mathematics 434 **Spring 2011** **Problem Sheet 7**

Due March 21

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
"*all ignorance toboggans into know and trudges up to ignorance again.*" – e.e.cummings, 1959

Problems

1. Do **one** of the following.
   (a) Determine the structure of the ring $\mathbf{Z}[x]/(x^2+3,p)$ where
      i. $p=3$
      ii. $p=5$
   (b) Describe the ring $\mathbf{Z}[i]/(2+i)$.
2. Describe the ring obtained from $\mathbf{Z}$ by adjoining an element α satisfying the two relations $2\alpha-6=0$ and $\alpha-10=0$.
3. Suppose we adjoin an element α to $\mathbf{R}$ satisfying the relation $\alpha^2=1$. Prove the resulting ring is isomorphic to the product ring $\mathbf{R}\times\mathbf{R}$,and find the element of $\mathbf{R}\times\mathbf{R}$ which corresponds to α.
4. Let α denote the residue of x in the ring $R'=\mathbf{Z}[x]/(x^4+x^3+x^2+x+1)$. Compute the expressions for $(\alpha^3+\alpha^2+\alpha)(\alpha+1)$ and α^5 in terms of the basis $(1,\alpha,\alpha^2,\alpha^3,\alpha^4)$.
5. Do **one** of the following.
   (a) In each case describe the ring obtained from $\mathbf{Z}$ by adjoining an element α satisfying the given relation.
      i. $\alpha^2+\alpha+1=0$
      ii. $\alpha^2+1=0$
   (b) Let $R=\mathbf{Z}/(10)$. Determine the structure of the ring R' obtained from R by adjoining element α satisfying each relation.
      i. $2\alpha-6=0$
      ii. $2\alpha-5=0$.
6. Describe the ring obtained from $\mathbf{Z}/12\mathbf{Z}$ by adjoining an inverse of 2.In particular, what 'standard' ring is isomorphic to this adjunction ring?
7. Let a be an element of a ring R, and let $R'=R[x]/(ax-1)$ be the ring obtained by adjoining an inverse of a to R. Prove that the kernel of the canonical map from R to R' is the set of elements $b\in R$ such that $a^nb=0$ for some $n>0$.
8. Let F be a field, t a free symbol and $R=F[t]$ the ring of polynomials on the variable t with coefficients in R. Adjoin an inverse to the variable t by forming the quotient $F[t,x]/(xt-1)$. Show that this ring is isomorphic to the ring $F[t,t^{-1}]$ of Laurent polynomials. A Laurent polynomial is a polynomial in t and t^{-1} of the form
$$f(t)=\sum_{i=-n}^{n}a_it^i=a_{-n}t^{-n}+\cdots a_{-1}t^{-1}+a_0+a_1t+\cdots+a_nt^n.$$