Mathematics 434 Spring 2011 Homework 04

Due February 21

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1. Count how many distinct ways there are to color the faces of a cube with six colors if each color is used exactly once?

   (a) Up to a rotation, how many ways can the faces of a cube be colored using three different colors?

2. Up to a rotation, how many different ways can the edges of a cube be colored using two colors?

3. A striped necktie has 12 bands of color. Each band can be colored by one of four possible colors. How many different-colored neckties are there ?

4. Let p be prime. Show that the number of different abelian groups of order p^n (up to isomorphism) is the same as the number of conjugacy classes in S_n .

5. Let G be a group with order p^n where p is prime and X be a finite G -set. If $X_G = \{x \in X : gx = x \text{ for all } g \in G\}$ is the set of elements fixed by the group action, then prove that $|X| \equiv |X_G| \pmod{p}$.

6. How many different arrangements of X's and O's are possible on a tic-tac-toe grid if two arrangements are considered the same when one is a rotation or reflection of the other. (Note that we want all arrangements – not just the ones that can occur when playing an actual game of tic-tac-toe.)