Mathematics 434 Spring 2011 Homework 03

Due February 17

Sylow Theorems

1. Do one of the following.

   (a) Prove no group of order p^2q, where p and q are prime, is simple.

   (b) Let G be a group of order $p^l m$. Our First Sylow theorem shows that G contains a subgroup of order p^l. On the other hand, our textbook (Judson) uses an induction proof to show that G contains a subgroup of order p^r for every integer $1 \leq r \leq l$. Adapt **our** proof to give a separate argument that G contains a subgroup of order p^r .

2. Do one of the following.

   (a) Prove the only simple groups of order less than 60 are groups of prime order.

   (b) Classify all groups of order 18.

3. Prove no group of order 224 is simple.