Mathematics 434 **Spring 2011** **Homework 02**

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

"*Personally, I'm always ready to learn, although I do not always like being taught.*" – Winston Churchill

Problems

1. Prove that the group $GL(2, \mathbf{F}_2)$ of invertible matrices with mod 2 coefficients is isomorphic to the symmetric group S_3.

2. Do **one** of the following.

   (a) Prove the formula $|G| = |Z(G)| + \sum |C|$ where the sum is over the conjugacy classes containing more than one element and $Z(G)$ is the center of G.

   (b) Rule out as many of the following as possible as Class Equations for a group of order 10.

      i. $1 + 1 + 1 + 2 + 5$

      ii. $1 + 2 + 2 + 5$

      iii. $1 + 2 + 3 + 4$

      iv. $1 + 1 + 2 + 2 + 2 + 2$

3. Determine the conjugacy classes in the group M of motions of the Euclidean plane.

4. Determine the order of the conjugacy class of $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ in $GL_2(\mathbf{F}_5)$. Here $\mathbf{F}_5$ is the field of integers modulo 5.

5. Let N be a normal subgroup of a group G. Suppose that $|N| = 5$ and that $|G|$ is an odd integer. Prove that N is contained in the center of G.