Mathematics 434 Spring 2011 Problems to Turn in: 01

February 3, 2011

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

"*Iron rusts from disuse; stagnant water loses its purity and in cold weather becomes frozen; even so does inaction sap the vigor of the mind.*" – Leonardo da Vinci

Problems

1. Prove the proposition.

2. Let S be a G-set and s an element of S. Let s' be an element of the orbit of s, say $s' = as$. Then

   (a) The set $T = \{g \in G : gs = s'\}$ is the left coset aG_s of the stabilizer of s in G.

   (b) The stabilizer $G_{s'}$ of s' in G is the conjugate subgroup $aG_s a^{-1}$ of the stabilizer of s in G.

3. Do **one** of the following.

   (a) Let $G = D_4$ be the dihedral group of symmetries of the square.

      i. What is the stabilizer of a vertex? Of an edge?

      ii. G acts on the set of two elements consisting of the diagonal lines. What is the stabilizer of a diagonal?

   (b) Let $G = GL\left(n, \mathbf{R}\right)$ operate on the set $S = \mathbf{R}^n$ by left multiplication.

      i. Describe the decomposition of S into orbits for this operation. That is, what are the distinct orbits?

      ii. What is the stabilizer of $\vec{e}_1$?

4. Do **one** of the following.

   (a) Let G be a group and let H be the cyclic subgroup generated by an element x of G. Show that if left multiplication by x fixes every coset of H in G, then H is a normal subgroup of G.

   (b) A map $\phi : S \to S'$ of G - sets is called a **homomorphism** of G- sets if $\phi\left(gs\right) = g\phi\left(s\right)$ for all $s \in S$ and all $g \in G$. Let ϕ be such a homomorphism. Prove the following.

      i. The stabilizer $G_{\phi(s)}$ contains the stabilizer G_s.

      ii. The orbit of an element $s \in S$ maps onto the orbit of $\phi\left(s\right)$.

5. Let G be the group of rotational symmetries of a cube C. Two regular tetrahedra Δ and Δ' can be inscribed in C, each using half of the vertices. What is the order of the stabilizer of Δ?