Spring 2010 **Exam 3** **Math** 300

April 20

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Name

Directions: Only write on one side of each page.

Do any (5) of the following

1. (20 points) Using any previous results, prove the uniqueness (but not the existence) portion of Proposition 4.3: Every segment has a unique midpoint.

2. (20 points) Using any previous results, prove Proposition 4.10: Hilbert's parallel postulate $\iff$ if k is parallel to l, m is perpendicular to k, and n is perpendicular to l, then either $m = k$ or $m \| l$.

3. (20 points) Using any results through Chapter 4, prove the following: Hilbert's parallel property holds $\iff$ if k, m, l are distinct lines, k is parallel to m, and m is parallel to l, then k is parallel to l.

4. (20 points) Using any results from neutral geometry, prove: Given parallel lines l and m. Given points A and B that lie on the opposite side of m from l; that is, if P is any point on l, A and P are on opposite sides of m and B and P are on opposite sides of m. Prove that A and B are on the same side of l.

5. (20 points) Using any results through Chapter 5, prove that Hilbert's parallel postulate implies Wallis' postulate. [Wallis' postulate is: Given any triangle $\triangle ABC$ and given any segment DE. There exists a triangle $\triangle DEF$ (having DE as one of its sides) that is similar to $\triangle ABC$.]

6. (20 points) Using any results through Chapter 6, prove the following in hyperbolic geometry. Let $\triangle ABC$ be any triangle and let L, M, and N be the midpoints of BC, AB, and AC, respectively. Use a proof by contradiction to prove that MN is **not** congruent to BL. [Hint: choose D so that $M * N * D$ and $ND \cong MN$ and show various triangles thus formed are congruent.]