Spring 2009 **Exam 1** **Math** 300

February 17

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Name

Directions: Only write on one side of each page.

Do any (5) of the following

1. From the homework exercises we know if M is a projective plane with only a finite number of points then every line has the same number of points incident with it. Call this number $n+1$. Prove that, in M, every point has at most $n+1$ lines incident with it.

2. Prove the following Proposition: Let l be a line in a projective plane M. If we denote the set of lines $\{m : m \text{ is incident with at least one point that lies on } l\}$ by L then the set L contains every line of M.

3. Use two models of incidence geometry to show that the following statement is independent of incidence geometry.

   (a) Given distinct lines l, m, and n.If l is parallel to m and m is parallel to n, then l is parallel to n.

4. Recall that a projective plane is a model of incidence geometry satisfying the elliptic parallel property and in which every line has at least three points incident with it.

   Let M be a projective plane and let M' be the interpretation of the undefined terms obtained by interpreting M' points to be the lines of M and interpreting the M' lines to be the points of M.

   (a) Cite results that show the interpretation M' is both a model of incidence geometry and satisfies the elliptic parallel property.

   (b) Complete the argument that M' is a projective plane by carefully proving every 'line' in M' is incident with at least three 'points'.

5. Using any previous results give a careful proof of Proposition 2.7.

   For every line l there are at least two distinct lines neither of which is l.

6. What is the smallest number of lines possible in a model of incidence geometry in which there are exactly 5 points? Include a careful argument supporting your claim (but you need not provide a formal proof.)