Figure 1:

Honors 213 — Spring 2009

Final Exam

May 13, 2009

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Name

Directions: Be sure to include in-line citations of pertinent theorems and propositions. Include any figures used in solving a problem. **Only write on one side of each page.**
"'Know thyself?' If I knew myself, I'd run away." – Johann von Goethe

Do any five (5) of the following.

Extra Credit Make a list of statements that hold in real Euclidean planes but do not hold in real hyperbolic planes. One point (up to 6) for each correct statement that is not in the list of ten in Exercise 1 of Chapter 6 in the textbook. One point off (down to 0) for each incorrect statement.

1. Show that any statement in the language of geometry that is a theorem in Euclidean geometry and whose negation is a theorem in hyperbolic geometry is equivalent to Hilbert's parallel postulate. Hint: Let N denote the axioms of neutral geometry, H Hilbert's parallel postulate and T the theorem in Euclidean geometry that is false in hyperbolic geometry. Then part of what we know is $(N \,\&\, H) \Longrightarrow T$. You are asked to show that given N then

$$T \Longleftrightarrow H.$$

2. Using any result through Meta Mathematical Theorem 1, prove the Corollary to Meta Mathematical Theorem 1.

   If Euclidean geometry is consistent then Hilbert's parallel property is independent of the axioms of neutral geometry.

3. Prove directly that Incidence Axiom 3 holds in the Poincaré Half-plane interpretation.

Figure 2:

4. Let l be a line in the Poincaré Disk model of hyperbolic geometry that is not a diameter of γ_P.Let P be a point interior to γ_P that is not incident with l. Construct one of the limiting parallel rays to l from P and explain why it satisfies the definition of limiting parallel ray. Specifically, describe the center and radius of the appropriate circle δ and explain why that circle satisfies all of the necessary properties.

5. Be sure to draw all sketches in this problem carefully. Let γ be a circle with center O and radius OR.

   (a) For the line l in the figure, draw (carefully sketch) the set of points l' that are the inverses with respect to γ of the points of l.

   (b) For the line l in the figure, draw (sketch) the set of points l' that are the inverses with respect to γ of the points of l.

   (c) For the line l in the figure, draw (sketch) the set of points l' that are the inverses with respect to γ of those points of l other than O.

   (d) For the circle δ in the figure, draw (sketch) the set of points δ' that are the inverses with respect to γ of the points of δ.

   (e) For the circle δ in the figure, draw (sketch) the set of points δ' that are the inverses with respect to γ of the points of δ. Note that δ is **not** orthogonal to γ.

   (f) For the circle δ in the figure, draw (sketch) the set of points δ' that are the inverses with respect to γ of the points of δ. Note that δ **is** orthogonal to γ.

6. Using any result previous to it, prove Theorem 6.2 (AAA)

   In a plane satisfying the acute angle hypothesis, if two triangles are similar then they are congruent.

Figure 3:

Figure 4:

Figure 5:

Figure 6:

Figure 7:

7. The circle-line continuity principle tells us that if a line passes through a point in the interior of a circle, then the line meets the circle in at least two points. Tighten this result by proving, in neutral geometry, that no line can meet a circle at more than two distinct points. You may use any result through Chapter 5.