6, 3, 1
4, 2, 1

5.4 & 5.5

Resources: Textbook, Solution Manual, TI-82

Ch. 5.4 p. 353 #74 $\lim_{x\to 0} \frac{1}{x^3}\int_0^x \frac{t^2}{t^4+1}\,dt$

$\lim_{x\to 0} \frac{\int_0^x \frac{t^2}{t^4+1}}{x^3} = \frac{0}{0}$ ⇐ Indeterminate form Use l'Hopital's Rule

$\lim_{x\to 0} \frac{\frac{x^2}{x^4+1}}{3x^2}$ ⇐ Use Fundamental Theorem of Calculus

$\lim_{x\to 0} \frac{x^2}{3x^2(x^4+1)}$

$\lim_{x\to 0} \frac{x^2}{3x^6+3x^2}$

$\lim_{x\to 0} \frac{2x}{18x^5+6x}$

$\lim_{x\to 0} \frac{2}{90x^4+6}$

$\frac{2}{6} = \boxed{\frac{1}{3}}$

The limit of $\frac{1}{x^3}\int_0^x \frac{t^2}{t^4+1}\,dt$ is equal to $\frac{1}{3}$. The limit was found by using the Fundamental Theorem of Calculus and l'Hopital's Rule.

Good

[redacted]

2. (5.5)

A) $\int \sqrt{1-x^2}\,dx = \int \cos^2(t)\,dt$ when $x=\sin(t)$

$= \int \sqrt{1-x^2}\,dx \quad x=\sin(t)$

$dx = \cos(t)\,dt$ /

$= \int \sqrt{1-(\sin(t))^2} \cdot \cos(t)\,dt$

$= \int \sqrt{1-\sin^2(t)} \cdot \cos(t)\,dt$

$= \int \sqrt{\cos^2(t)} \cdot \cos(t)\,dt$

$= \int \cos(t) \cdot \cos(t)\,dt$

$= \int \cos^2(t)\,dt$ ✓ /

By substituting sin(t) in for x and then distributing the squared term to the outside ($\sin^2(t)$), we get a trig function under the radical. $1-\sin^2(t) = \cos^2(t)$ which came from $\cos^2(t) + \sin^2(t) = 1$. This simplifies our equation and allows the square root to cancel with the squared cosine term. using the cos(t) dt function as a substitute for dx, the function works itself out into $\int \cos^2(t)\,dt$. /
(university calculus)

B) $\int \cos^2(t) \neq \frac{1}{3}\cos^3(t) + C$

If we take the derivative of $\frac{1}{3}\cos^3(t) + C$ we should expect to get something not equal to $\cancel{\int}\cos^2(t)$ ~~dt~~ T: /

$F(t) = \frac{1}{3}\cos^3(t) + C$ ← constant will go to zero.

$f'(t) = \frac{3}{3}\cos^2(t) \cdot -\sin(t)$ /

$f'(t) = \cancel{\int} -\cos^2(t)\sin(t) \neq \cancel{\int}\cos^2(t)$ ✓

T: There should not be an integral sign.

The integral of $\int \cos^2(t)\,dt$ doesn't equal $\frac{1}{3}\cos^3(t)$ because you have to use chain rule if you work backwards and take the derivative of $\frac{1}{3}\cos^3(t)$. when you do this, you get $\int -\cos^2(t)\sin(t)$ which is not the same as $\int \cos^2(t)$. (university calculus) /

Good