Mathematics 290-A **Exam 1** **Spring 2007**

January 30, 2007

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Name

Technology used: ____

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do BOTH of these "Computational" Problems

C.1. Solve the following system of linear equations by hand. Write the solution set using column vector notation. Make sure you copy the equations correctly.

$$\begin{aligned} x_4 + 2x_5 - x_6 &= 2 \\ x_1 + 2x_2 + x_5 - x_6 &= 0 \\ x_1 + 2x_2 + 2x_3 - x_5 + x_6 &= 2 \\ x_1 + 2x_2 + 2x_3 + x_4 + x_5 &= 4 \end{aligned}$$

C.2. Find a 4×5 matrix A, that is **not** in reduced row-echelon form, whose null space is the set

$$\left\{ \begin{bmatrix} 2x_2 - 6x_4 \\ x_2 \\ -5x_4 \\ x_4 \\ 7x_2 + x_4 \end{bmatrix} : x_2,\ x_4 \in \mathbf{C} \right\}$$

Do Two (2) of these "In text, class or homework" problems

M.1. Suppose A and B are $m \times n$ matrices. Give a detailed explanation of why if A is row-equivalent to B then B is row-equivalent to A.

M.2. Suppose that B is an $m \times n$ matrix in reduced row-echelon form. Build a new, likely smaller, $k \times l$ matrix C as follows. Keep any collection of k adjacent rows, $k \leq m$. From these rows, keep columns 1 through l, $l \leq n$. Prove that C is in reduced row-echelon form.

M.3. Prove that a system of linear equations is homogeneous if and only if it has the zero vector as a solution.

Do BOTH of these "Not in Text" problems

T.1. Suppose that A is the coefficient matrix of a consistent linear system of equations and that two of the columns of A are identical. Prove that there must be an infinite number of solutions to the system of equations.

T.2. Let A be a 4×4 matrix and let $\vec{b}$ and $\vec{c}$ be vectors of constants with 4 entries each.

(a) If we are told that the linear system of equations $LS\left(A, \vec{b}\right)$ has a unique solution, what can you say about the solutions set of $LS\left(A, \vec{c}\right)$? Why?

(b) If we are told that the linear system of equations $LS\left(A, \vec{b}\right)$ is inconsistent, what can you say about the solutions set of $LS\left(A, \vec{c}\right)$? Why?

(c) Now suppose B is a 4×3 matrix and we know that $LS\left(B, \vec{b}\right)$ has a unique solution. What can you say about the solution set of $LS\left(B, \vec{c}\right)$? Why?