Mathematics 181 Spring 2007 Project 6

Due April 27, 2007

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Name

Only write on one side of each page.
Be sure to re-read the WRITING GUIDELINES rubric, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution.**
"*Do not imagine that Mathematics is hard and crabbed, and repulsive to common sense. It is merely the etherealization of common sense.*" – Lord Kelvin

Integration using Polar Coordinates

Do both of the following.

1. In our textbook the authors claim that substituting $x = f(\theta)\cos(\theta)$ and $y = f(\theta)\sin(\theta)$ into the formula for the length of a parametrized curve $L = \int_\alpha^\beta \sqrt{(dx/d\theta)^2 + (dy/d\theta)^2}\, d\theta$ yields the formula (involving only r and θ and not x or y) $L = \int_\alpha^\beta \sqrt{r^2 + (dr/d\theta)^2}\, d\theta$. Carefully explain why this is true.

2. If f is continuous, the average value of the polar coordinate r over the curve $r = f(\theta)$, $\alpha \le \theta \le \beta$, with respect to θ is given by
$$r_{avg} = \frac{1}{\beta - \alpha}\int_\alpha^\beta f(\theta)\, d\theta.$$
Use this formula to find the average value of r with respect to θ over the following polar curves (here a is a positive constant). In each case include a careful sketch of the curve (use $a = 2$) and briefly explain why the answer you get "makes sense".