Math 181 **Spring 2007** **Project 04**

Due March 9

____
Name

Be sure to re-read the WRITING GUIDELINES rubric, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution.**

"*Without education we are in a horrible and deadly danger of taking educated people seriously.*" (G.K. Chesterton)

Signal Reconstruction – A Beginning (How does the music get off the CD?)

Basic Background

When we hear music the tympanic membrane in our ears (the eardrum) vibrates because of changes in air pressure due to pressure waves produced by movement of a similar membrane in our stereo speakers. These pressure waves are described mathematically as the sum of sine and cosine functions that have different amplitudes and frequencies. So, phrased in a simplified form, a continuous function that is the sum of all of the sine and cosine functions describing the tones of what we hear (this continuous function is called a "signal") is produced by your stereo system and is used to move the speaker membrane to produce the pressure waves we "hear". In order for this to happen several complex things happen. The one associated with this project involves the sampling of the continuous function that describes the music signal.

The outputs of this continuous function are recorded ("sampled") at a finite number of times and the samples digitized so they can be recorded on a glass master that is used to produce compact disks. [The making of a glass master is a highly technical process where the digital information from the source is transferred in "PIT" form onto a polished glass that is covered with a photo resistant coating. The term "PIT" refers to a series of microscopic imprints that are formed on the photosensitive layer that coats that glass. As a laser beam comes in contact with the coated glass, it translates the digital information to the coating on the glass in the form of pits.]$_1$ There is much interesting mathematics behind the fact that, if one samples the **continuous** function at evenly spaced times that occur at least twice as often as the maximum frequency of all the sine and cosine functions whose sum makes up the signal then one can use that finite (and hence easily digitized) collection of samples to completely reconstruct the **continuous** signal. The mathematics used in this process involves discrete mathematics (because of the discrete nature of the samples) that we don't quite have the proper background to understand (when we finish Chapter 8 we will be better prepared.) However, the general mathematical processes used in this discrete mathematics have a continuous analogy where one uses definite integrals in a fashion for which we are completely prepared. This project is about that continuous analog.

Project Background

Any music that has been sampled and recorded on a compact disk has only a finite number of frequencies. All of the extremely high and extremely low frequencies are stripped off of the signal by the sampling process. [This is why there are music aficionados who pay $30,000 for turntable based music systems. Those old style records contain many more of the very high and low frequencies and these aficionados say they can hear the difference. However there are a number of studies that show that **most** people cannot distinguish the difference. I do not know who funded those studies ;-).] That these frequencies are stripped out means that when we work to reconstruct the continuous signal, we know exactly what frequencies are left over to occur in the sum of sine and cosine functions that make it up. One fact that can be shown, but that we will not show in this project, is that if we are reconstructing the continuous signal over a known time interval $0 \leq t \leq T$, then there is a "fundamental frequency, $f_0 = \frac{1}{2T}$ with the property that every sine and cosine function in the sum of functions that make up our signal has a frequency that is an integer multiple of f_0. Hence the continuous signal, which we denote by $p(t)$, we are going to reconstruct can be written in the form

$$p(t) = \sum_{k=1}^{N} \left[A_k \cos(2\pi k f_0 t) + B_k \sin(2\pi k f_0 t)\right].$$

where T, N and f_0 are known beforehand but the amplitudes $A_1, A_2, \cdots, A_N$ and $B_1, B_2, \cdots, B_N$ are not. So, all we need to know in order for us to determine which sine and cosine functions make up the continuous signal $p(t)$ (which we need to know in order to reconstruct it) are these amplitudes. Seeing how integrals are used in doing this is the purpose of this project.

The Project Itself

1. Compute the two integrals $\int_0^T \cos(2\pi m f_0 t)\cos(2\pi n f_0 t)\ dt$ and $\int_0^T \sin(2\pi m f_0 t)\sin(2\pi n f_0 t)\ dt$ where m and n are **different** integers. Show your work.
2. Use the above computations to determine all of the amplitudes A_k, $1 \leq k \leq N$ and B_k, $1 \leq k \leq N$.
3. Hints supplied on request.

Sources

1. http://www.emimusic.ca/making_cd.asp

- http://www.st-andrews.ac.uk/~jcgl/Scots_Guide/iandm/part7/page3.html
- http://en.wikipedia.org/wiki/Nyquist%E2%80%93Shannon_sampling_theorem
- http://cnx.org/content/m10791/latest/