## Math 258 Spring 2004 – Final Exam

Name\_\_\_\_\_

Show your work. This test is open book, open notes. Pay careful attention to notation. 11 points per problem

- 1. Let  $f(x) = x^2 3x$
- a) Find the difference quotient  $\frac{f(x+h) f(x)}{h}$

b) Find 
$$f'(x)$$
 by finding  $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ 

2. Find the first derivative:

a) 
$$f(x) = \frac{5}{7x^2 + 1}$$

b) 
$$f(x) = 3x\sqrt{2x^2 + 3}$$

c) 
$$f(x) = 10e^{x^2 + 3x}$$

d) 
$$f(x) = \ln\left(\frac{1}{x^2}\right)$$

Use the following information for problems 3-4: The temperature (in °F) in a greenhouse varies during the day. On one particular spring day, the temperature followed this pattern:

$$T(x) = 30 + 3x + x^2 - \frac{x^3}{10}$$

where **x** is the number of hours after 8AM ( $0 \le x \le 12$ )

3. a) What was the initial temperature in the greenhouse (at 8AM)?

b) How fast was the temperature changing at 10AM?

4. a) When did the temperature reach a maximum?

b) What was the maximum temperature reached?

c) What was the minimum temperature reached?

Use the following information for problems 5 and 6: A snowboard manufacturer produces two types of snowboard: Type X and Type Y. The total cost of producing these snowboards (in millions of \$) is:

$$C(x, y) = x^{2} - 2xy + 2y^{2} + 6x - 9y + 5$$

Revenues from the sale of these snowboards (in millions of \$) is:

R(x, y) = 2x + 3y

where x is the number of Type X snowboards produced (in thousands) and y is the number of Type Y snowboards produced (in thousands). Recall that the profit from the sales of these snowboards is

P(x, y) = R(x, y) - C(x, y)

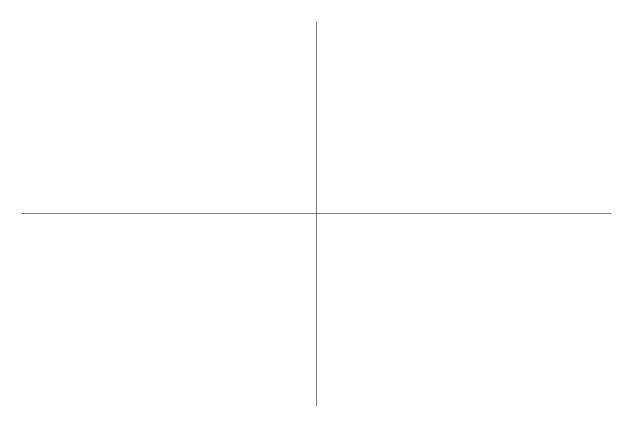
5. a) How many Type X snowboards and Type Y snowboards should the company produce to maximize profits?

b) What is the maximum profit?

6. The manufacturer's factory can produce up to 10,000 snowboards in a year. Use the technique of Lagrange Multipliers to determine how many Type X snowboards and Type Y snowboards the company produce should produce to maximize profit if total production is 10,000 snowboards.

7. A ship uses  $5x^2$  dollars of fuel per hour when traveling at a speed of x miles per hour. The other expenses of operating the ship are \$1500/hour. What speed minimizes the cost of a 1000 mile trip?

8. Consider the function  $f(x) = x^4 - 2x^3$ . Without using your graphing calculator, sketch the graph of f(x). Identify relative maxima and minima and all points of inflection.



9. Find:

a) 
$$\int \left(\frac{1}{2}x^2 + 3x\right) dx$$

b) 
$$\int_{1}^{4} \frac{1}{x^3} dx$$

c) The area of the region bounded by the graphs of

 $f(x) = x^2$  and  $g(x) = -x^2 + 8$ 

Extra Credit (1 point!): Who won the Super Bowl in 2004?

**New England Patriots** 

**New England Patriots** 

I have no idea, but I'll guess: New England Patriots

## Have a good summer!