Math 258 Spring 2004 – Final Exam

Name________________________________

Show your work. This test is open book, open notes. Pay careful attention to notation. 11 points per problem

1. Let $f(x) = x^2 - 3x$

a) Find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

b) Find $f'(x)$ by finding $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

2. Find the first derivative:

a) $f(x) = \dfrac{5}{7x^2 + 1}$

b) $f(x) = 3x\sqrt{2x^2 + 3}$

c) $f(x) = 10e^{x^2 + 3x}$

d) $f(x) = \ln\left(\dfrac{1}{x^2}\right)$

Use the following information for problems 3-4: The temperature (in °F) in a greenhouse varies during the day. On one particular spring day, the temperature followed this pattern:

$$T(x) = 30 + 3x + x^2 - \frac{x^3}{10}$$

where x is the number of hours after 8AM ($0 \le x \le 12$)

3. a) What was the initial temperature in the greenhouse (at 8AM)?

b) How fast was the temperature changing at 10AM?

4. a) When did the temperature reach a maximum?

b) What was the maximum temperature reached?

c) What was the minimum temperature reached?

Use the following information for problems 5 and 6: A snowboard manufacturer produces two types of snowboard: Type X and Type Y. The total cost of producing these snowboards (in millions of $) is:

$$C(x, y) = x^2 - 2xy + 2y^2 + 6x - 9y + 5$$

Revenues from the sale of these snowboards (in millions of $) is:

$$R(x, y) = 2x + 3y$$

where x is the number of Type X snowboards produced (in thousands) and y is the number of Type Y snowboards produced (in thousands). Recall that the profit from the sales of these snowboards is

$$P(x, y) = R(x, y) - C(x, y)$$

5. a) How many Type X snowboards and Type Y snowboards should the company produce to maximize profits?

b) What is the maximum profit?

6. The manufacturer's factory can produce up to 10,000 snowboards in a year. Use the technique of Lagrange Multipliers to determine how many Type X snowboards and Type Y snowboards the company produce should produce to maximize profit if total production is 10,000 snowboards.

7. A ship uses $5x^2$ dollars of fuel per hour when traveling at a speed of x miles per hour. The other expenses of operating the ship are $1500/hour. What speed minimizes the cost of a 1000 mile trip?

8. Consider the function $f(x) = x^4 - 2x^3$. Without using your graphing calculator, sketch the graph of $f(x)$. Identify relative maxima and minima and all points of inflection.

9. Find:

a) $\int \left(\frac{1}{2} x^2 + 3x \right) dx$

b) $\int_1^4 \frac{1}{x^3} dx$

c) The area of the region bounded by the graphs of

$f(x) = x^2$ and $g(x) = -x^2 + 8$

Extra Credit (1 point!): Who won the Super Bowl in 2004?

New England Patriots ______

New England Patriots ______

I have no idea, but I'll guess: New England Patriots ______

Have a good summer!