Math 258 – First Hour Exam – Spring, 2004 v1

Name _____

Show your work. Partial credit will be given where appropriate. 16 points per problem
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1. Differentiate:

$f(x) = x^8 + 3x^2 + 5$

$f(x) = \sqrt{x^2 + 1}$

$f(x) = (5x + 1)^4$

2. Let $f(x) = x^2 + 2$

a) Find the difference quotient $\frac{f(x+h) - f(x)}{h}$

b) Find $f'(x)$ by finding $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

3. Let $y = \frac{1}{3x}$

Find:

$\frac{dy}{dx} =$

$\left.\frac{dy}{dx}\right|_{x=5} =$

$\frac{d^2y}{dx^2} =$

$\left.\frac{d^2y}{dx^2}\right|_{x=5} =$

4. Evaluate the following limits:

$\lim_{x\to 2} \frac{x^2+1}{5-x}$

$\lim_{x\to 3} \frac{x^2-x-6}{x-3}$

$\lim_{x\to 7} \frac{x^2}{x-7}$

5. A Super Bowl fan, momentarily distracted during the halftime show, drops a rubber ball down an access ramp at the stadium. As the ball rolls down the ramp, its distance from the fan after *t* seconds is: $s(t) = t^3 + 2t^2$ feet.

a. How far has the ball rolled at 3 seconds?

b. How fast is the ball rollling at 3 seconds?

c. How fast is the ball's velocity changing at 3 seconds?

6. Let $f(x) = \frac{2}{3}x^3 - 2x^2 - 6x + 6$

There are two points at which the tangent line to the graph of this function has a slope of –6. Find those points.

Extra credit (5 points): Is $f(x) = \begin{cases} x-1 & ;0 \le x < 1 \\ 1 & ; x = 1 \\ 2x-2 & ; x > 1 \end{cases}$ continuous at x=1? Why or why not?