## Due January 27

## Name

Be sure to re-read the WRITING GUIDELINES rubric, since it defines how your project will be graded. In particular, you may discuss this project with others but you may not collaborate on the written exposition of the solution.
"It is by logic that we prove but by intuition that we discover." (Henri Poincaré)

## Two Theorems to Prove

Part 1. There is exactly one real number that can be written in the position marked byin Theorem (1) below that makes the statement true. Determine that number and prove the resulting theorem.

Theorem 1 (1) Given any two solutions $\left(\beta_{1}, \beta_{2}, \cdots, \beta_{n}\right)$ and $\left(\gamma_{1}, \gamma_{2}, \cdots, \gamma_{n}\right)$ of the linear equation $a_{1} x_{1}+$ $a_{2} x_{2}+\cdots+a_{n} x_{n}=b$.If $\left(\beta_{1}+\gamma_{1}, \beta_{2}+\gamma_{2}, \cdots, \beta_{n}+\gamma_{n}\right)$ is also a solution. Then $b=\square$.

Part 2. Replace the $\square$ in Theorem (2) with the same number you used in Part 1 and then prove Theorem (2)

Theorem 2 (2) Given any two solutions $\left(\beta_{1}, \beta_{2}, \cdots, \beta_{n}\right)$ and $\left(\gamma_{1}, \gamma_{2}, \cdots, \gamma_{n}\right)$ of the linear equation $a_{1} x_{1}+$ $a_{2} x_{2}+\cdots+a_{n} x_{n}=b$.If $b=\square$. Then $\left(\beta_{1}+\gamma_{1}, \beta_{2}+\gamma_{2}, \cdots, \beta_{n}+\gamma_{n}\right)$ is also a solution.

Part 3. Do these two theorems say the same thing about solutions of linear equations? Why or why not?

