Mathematics 221 Spring 2004 Project 2

Due: Not Collected

Collaborators **Name**

Directions: Be sure to follow the guidelines for writing up projects as specified in the course information sheet (passed out on the first day of class). Whenever appropriate, use in-line citations, including page numbers and people consulted when you present information obtained from discussion, a text, notes, or technology. **Only write on one side of each page.**

"Mathematicians do not study objects, but relations among objects; they are indifferent to the replacement of objects by others as long as relations do not change. Matter is not important, only form interests them."
— Henri Poincaré

Project Description

For this project please submit your efforts on exactly one of the following. (However, you should be able to do every problem in the list.)

1. Suppose $\mathbf{a}$ and $\mathbf{b}$ are given non-zero vectors such that $\mathbf{a} \cdot \mathbf{b} = 0$, k is a given scalar, and $\mathbf{x}$ and $\mathbf{y}$ are unknown vectors satisfying

   (a) $2\mathbf{x} + \mathbf{y} = \mathbf{a}$

   (b) $\mathbf{x} \times \mathbf{y} = \mathbf{b}$

   (c) $\mathbf{x} \cdot \mathbf{a} = k$.

   Use vector algebra to find $\mathbf{x}$ and $\mathbf{y}$ in terms of $\mathbf{a}, \mathbf{b}$, and k. [You will need to explain to your audience when you use the various properties of the dot and cross products.]

2. Given two distinct lines in three-dimensional Euclidean space exactly one of the following three things must happen: the lines are parallel in the sense that they have parallel direction vectors, or the lines meet at exactly one point, or the lines are **skew** in that they are neither parallel nor meet.

   (a) In which, if any, of these three cases is there a single plane containing both lines? Give a geometric explanation.

   (b) Determine if the following two lines are parallel, skew, or meet in a single point and, if they lie in a single plane find an equation of that plane.

$$\begin{aligned} \overrightarrow{r}(t) &= \quad < 1, 1, 0 > + t < 1, -1, 2 > \\ \overrightarrow{r}(t) &= \quad < 4, -2, 2 > + t < -1, 1, 0 > \end{aligned}$$