Honors 213 **Spring 2003** **Exam 1**

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

Problems

1. (10 points) Negate the following logical statement to the point there is no longer an implication.

$$\forall \varepsilon\ \exists N \quad (n > N) \Longrightarrow (|a_n - L| < \varepsilon)\,.$$

2. (10 points) Prove the following logical statement is a tautology.

$$(\sim q \wedge (p \Longrightarrow q)) \Longrightarrow \sim p$$

3. (20 points) Using the Axioms and previous results to justify every step, formally prove Proposition 2.4.

   For every point there is at least one line passing through it.

4. (15, 5 points) Here is an interpretation of the undefined terms of incidence geometry: Fix a circle in the Euclidean plane. Interpret "point" to mean an ordinary Euclidean point *inside* the circle. Interpret "line" to mean a chord of the circle. Let 'incidence" mean that the point lies on the chord in the usual Euclidean sense.

   (a) Which of the axioms of Incidence geometry are satisfied by this interpretation? Explain.

   (b) Does this interpretation have a parallel property? If so, is it the elliptic, Euclidean, or hyperbolic parallel property? Explain.

5. (20 points) Recall that a projective plane is a model of incidence geometry satisfying the elliptic parallel property and in which every line has at least three points incident with it.

   Let M be a projective plane and let M' be the interpretation of the undefined terms obtained by interpreting M' points to be the lines of M and interpreting the M' lines to be the points of M.

   (a) Prove that M' satisfies Incidence Axiom 2.

   (b) Prove that M' satisfies Incidence Axiom 1.

   (c) Prove that M' satisfies the elliptic parallel property.

6. Do **one** of the following.

   (a) (20 points) Modelling problem:

      i. Explain how one uses models of an axiomatic system to prove a given statement is **independent** of that axiomatic system.

      ii. Use two models of incidence geometry to show that the following statement is independent of incidence geometry.
      Given distinct lines l, m, and n.If l is parallel to m and m is parallel to n, then l is parallel to n.

   (b) (20 points) What is the smallest number of lines possible in a model of incidence geometry in which there are exactly 5 points? Include a careful argument supporting your claim (but you need not provide a formal proof.)