January 22

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Name

"Pure mathematics consists entirely of such asservations as that, if such and such a proposition is true of anything*, then such and such another proposition is true of that thing. It is essential not to discuss whether the first proposition is really true, and not to mention what the anything of which it is supposed to be true .... If our hypothesis is about* anything *and not about some one or more particular things, then our deductions constitute mathematics. Thus mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true."* – Bertrand Russell
"Mathematics is the language with which God has written the universe" -Galileo Galilei, physicist and astronomer (1564-1642)

Problems

If possible, give examples of sets A, B such that

1. (a) $A \subset B$
   (b) $A \subsetneqq B$
   (c) $A \in B$
   (d) $A \notin B$
   (e) $A \subset A$
   (f) $A \subsetneqq A$
   (g) $A \notin A$
   (h) $A \in A$

2. Write out all the subsets of the following sets. Which are not proper subsets? Guess a general formula for the number of distinct subsets of a given, finite, set. Justify your formula is correct with a reasonable argument (it need not be a formal proof).

   (a) $A = \{1\}$, $B = \{a, b\}$, $C = \{x, y, z\}$

3. If the universal set is the set of all real numbers, find the set described by

   (a) $\{x|x \text{ is an integer}\} \cap \{x|x > 0\}$
   (b) $\{x|x \text{ is a negative integer}\} \cup \{x|x \text{ is a positive integer}\}$

4. Show that

   (a) if $A \subset B$, then $A \cap B = A$
   (b) if $A \subset B$,then $A \cup B = B$

5. Look up the definition of uniform continuity of a function on a set, explain what it means in your own words, convert it into logical notation, write out its negation in logical notation, and describe this negation in your own words.

6. Show that for any set (including infinite sets) A it is not the case that A is in one-one correspondence with $P(A)$.