1 Problem Set 1

1.1 Rubik's Cube Problems.

1. Show that any subgroup H of S_n that contains at least one odd permutation has exactly the same number of odd permutations as even permutations.

2. Show that

   (a) S_n is generated by the set of transpositions: $(1,2), (1,3), \cdots, (1,n)$.

   (b) A_n is generated by the set of all 3 - cycles (or, show A_n is generated by the set of all disjoint pairs of transpositions.)

3. Find the cycle representations of UR

4. Using the F^2R^2 subgroup as your inspiration, find processes of the Rubik's Cube that have the effect of

   (a) swapping the ul cubelet with the dl cubelet and the ub cubelet with the db cubelet.

   (b) swapping the ur cubelet with the dr cubelet and the ub cubelet with the db cubelet.

   (c) swapping the uf cubelet with the df cubelet and the ul cubelet with the dl cubelet.

   (d) swapping the fl cubelet with the bl cubelet and the uf cubelet with the bf cubelet.

   (e) swapping the fr cubelet with the br cubelet and the df cubelet with the db cubelet.

   (f) swapping the fr cubelet with the fl cubelet and the dr cubelet with the dl cubelet.]

5. The process $(F^2R^2)^3$ swaps two pairs of edge cubelets (it swaps the uf cubelet with the df cubelet and the ur cubelet with the dr cubelet.) What effect does conjugation of this element have on the cube? Spefically, what are the effects of YXY^{-1} where $X = (F^2R^2)^3$ and Y ranges over F, B, L, R, U, D and the inverses of these six motions?

6. Using the solution to the previous problem, describe an algorithm for interchanging any two pairs of edge cubelets.